

## HAS MORE INDEPENDENCE AFFECTED BANK OF ENGLAND'S REACTION FUNCTION UNDER INFLATION TARGETING? LESSONS FROM TAYLOR RULE EMPIRICS

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**ABSTRACT.** This paper is an empirical investigation into the question of whether increased independence affects central bank behavior, in particular when monetary policy is already in an inflation targeting regime. We take advantage of the unique experience in that sense of the United Kingdom, where the Bank of England was granted operational independence from Her Majesty's Treasury only in May 1997, while inflation targeting had been implemented since October 1992. Our strategy is to estimate Taylor rules employing alternative specifications, econometric methods and variable proxies in search for robust results that survive most of those modifications. The key lesson we extract from UK quarterly data is that the Bank of England has responded to the output *gap*, and not at all to output *growth*, the more so *after* receiving operational independence, when the gap has been positive or close to zero and inflation credibly stabilized at target. We find no unambiguous evidence for any definite change in the Bank's reaction to inflation or in the degree of its interest rate smoothing. Our main import is to argue that both the asymmetry of the monetary policy reaction function across the cycle and the response to the output gap, not growth, are fully consistent with New Keynesian theory, especially under inflation targeting. Anchored inflation and economic expansion during the post-independence period thus complement greater autonomy in influencing the behavior of the Bank of England, yet clear separation of the individual contribution of each of these effects appears challenging given our short sample.

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## 1. THE RECENT UK MONETARY FRAMEWORK AS AN 'ECONOMIC EXPERIMENT'

This paper makes an attempt to gain insights from a type of institutional change that arises only rarely in economic contexts. More precisely, our objective here is to address the question whether, and how, increased independence affects central bank behavior. We examine this issue under inflation targeting, the monetary strategy that has become dominant in the modern world, and in terms of empirically recovered policy reaction functions. We thus want to see what lessons can be learnt from a regime *shift*, to central bank operational independence, within a *stable* broader policy framework, of inflation-forecast targeting. This particular 'economic experiment' has been reflected in the recent experience with monetary policy making in the United Kingdom (UK). UK monetary authorities moved to inflation(-forecast) targeting in October 1992, and in May 1997 the Bank of England (BoE) was formally granted operational independence from Her Majesty's (HM) Treasury. Because of the explicit public announcement of the timing of these changes, toward inflation targeting and toward operational independence, both could be interpreted as *exogenous*. We therefore see in that a rare opportunity to explore and disentangle – to the extent possible – the effect(s) of increased central bank independence on (i.e., in addition to) policy responses under inflation targeting. To the best of our knowledge, this is the first paper to directly address the interesting issue summarized in the title.

There are, of course, huge literatures on both central bank independence and inflation targeting, but in isolation. We believe that the contribution of the present econometric study is to look at their *intersection*. In a broader sense, theoretical and empirical models of monetary policy are abundant as well. The same goes for a particular subset of these models, namely those focusing on simple instrument, or Taylor, rules – due to Taylor (1993) who proposed and illustrated such a rule on data for the United States (US) – describing monetary policy, also referred to as central bank feedback rules or reaction functions. To have a method of measurement and analysis that is – as it should be in an initial paper on a topic – enough straightforward and objective, we opt here for an application of Taylor rules to UK inflation targeting data. Extensions to a richer methodology or to other country cases in similar monetary circumstances thus remain for further research. Our sample for the present study consists of *quarterly* observations, mostly because GDP-related data, used to measure the output gap term in Taylor rules, are much more precise at a quarterly frequency.<sup>1</sup> It purposefully begins in October 1992, when inflation targeting was introduced in the UK, to be split in two subsamples in May 1997, when operational independence was granted to the Bank of England, thus covering completely the Bank's *pre*- and *post*-independence periods of inflation targeting until now. We essentially build on Clarida, Gali and Gertler (1997, 1998 a, b, 1999, 2000) and the broader empirical (and theoretical) literature on monetary policy reaction functions within the New Keynesian macromodel but also on Nelson (2000, 2001, 2003) and the relevant Taylor rule studies on the UK quoted further down.

We are well aware of the major criticisms concerning the Taylor rule approach to monetary policy interpretation. It is true that (i) such single-equation techniques are *overly simplistic*; yet they represent a good initial benchmark in summarizing the outcomes of monetary policy, which can then be cross-checked by a more complete system estimation (e.g., VAR methods). It is true as well that (ii) Taylor rules are *often unstable*, the more so over longer time spans and with more structural breaks in the data; but this is, in fact, duly reflected in our motivation for the present paper and addressed in it. That is particularly why we take advantage of the 'economic experiment' provided by the UK inflation targeting experience, and we do so in relatively short subsamples: *before* and *after* the publicly announced major policy change, namely the granting of operational independence to the Bank of England in May 1997, with the broader Bank monetary strategy remaining otherwise essentially unchanged. Finally, no matter the original Taylor (1993) rule and its well-known good visual fit to US post Bretton Woods data, the literature that followed has investigated (iii) a number of varieties of central bank reaction functions. These include, as we shall discuss: (a) other variables, notably the lagged dependent variable (interpreted as interest rate smoothing), the exchange rate (explicitly) and the price level (in the so-called hybrid rules) in addition to the inflation rate and the output gap (or real output growth, as preferred by certain authors); (b) backward-looking (adaptive) behavior and forward-looking (rational) expectations formation of economic agents – notably, the central bank – involving shorter or longer horizons (lag/lead structure); (c) higher (i.e., monthly) data frequency; (d) nonlinear functional forms.

<sup>1</sup>However, it may be desirable to revisit the robustness of our findings here with *monthly* data as well. Monthly data will allow for a three times higher number of observations per (sub)sample, and will thus very likely increase the quality of the statistical output. Moreover, the Monetary Policy Committee (MPC) of the Bank of England meets to decide on its interest rate instrument every month. And, finally, by now there are reliable monthly indicators of GDP in most industrialized countries, including the UK.

To reduce this diversity of feedback equations as potential descriptions of monetary policy in the context we are interested in, we first of all limit attention to the period of UK inflation targeting only, 1992-2004. We then allow for several versions of the estimated Taylor rules, selecting them with view to their relevance to our methods of econometric estimation and to the particular country, the UK, we study here. It is interesting to note that we rely on *instrument* rules of the Taylor type to capture (ex post) the essential features of monetary policy in the UK, the latter being normatively defined (ex ante) by a *targeting* rule with respect to inflation forecasts. We duly clarify the slight difference implied by this terminology later on and argue that our methodology would not be incorrect, even in the British institutional context, as a positive description of Bank of England's actual monetary policy.

In a preview of our main findings, classic Taylor rules perform in an impressive way in describing UK monetary policy throughout the period of operational independence when estimated by Ordinary Least Squares (OLS) and Two-Stage Least Squares (TSLS); moreover, they indicate that the Bank of England has definitely responded to the output gap after – but not before – receiving greater autonomy. However, because of the well-known critique to such contemporaneous specifications, we place predominant weight on our empirical results from forward-looking Taylor rules incorporating interest rate smoothing based on the Generalized Method of Moments (GMM). These results confirm what OLS and TSLS have partly uncovered earlier. Our key contribution consists in finding that during inflation targeting (i) the Bank of England has reacted to the output *gap* but *not* to output *growth* and that (ii) it has done so in an *asymmetric* way: under conditions of economic expansion near to and beyond (the available estimates of) potential output, as those having prevailed in the *post*-independence subsample, the Bank has responded *much more aggressively* relative to the *pre*-independence subsample, when the British economy has been (farther) below from potential. We thus present evidence that the Bank's *de facto* behavior could be characterized as *flexible* inflation targeting, i.e., one also paying attention to the output gap in addition to inflation, as is imposed on the Bank *de jure*, and not *strict* inflation targeting. As for the *magnitude* of the reaction to inflation and the *degree* of interest rate smoothing, we do not find overwhelming evidence to be able to conclude in favor of any important, substantial change across our two subsamples. Finally, it proves difficult to disentangle the effect(s) of operational independence from those of the business cycle and of the low inflation stabilized at target, and future research will be needed.<sup>2</sup>

The paper is further down structured as follows. In the next section we first summarize the minimum required theoretical and empirical background that we need to introduce the literature in a special-purpose notation and to justify our econometric approaches later on. Section 3 then describes the data and some preliminary tests. Section 4 presents our alternative estimation methods and specifications of Taylor rules, discussing the numerous econometric results we obtain and offering a unifying interpretation of our principal findings, and section 5 concludes. Sections 6 and 7 serve the role of appendices (referred to as A and B, respectively): Appendix A documents the most important features of our data and econometric results in tables and figures; Appendix B contains, in turn, a detailed analytical derivation of the formulas we used for the (approximate) computation of standard errors for most policy response coefficients based on the delta method.

## 2. A TAXONOMY OF TAYLOR RULES IN A UNIFORM NOTATION

**2.1. Origins of Monetary Rules.** Woodford (2003), chapter 1, traces the intellectual history of policy reaction functions back to the works Wicksell (1898, 1907).

**2.1.1. The Wicksellian Rule.** Wicksell (1898), as quoted in Woodford (2003), advocated not only a fiat money regime for the world as a whole (in place of the then existing gold standard) but also price-level targeting as a preferable monetary strategy. Although Wicksell expressed such ideas in words, Woodford (2003), p. 38, interprets them mathematically in the form of what he sometimes calls a simple *Wicksellian* (interest rate) rule for the management of such a fiat monetary system:

$$i_t^T = \bar{i} + \phi p_t,$$

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<sup>2</sup>A companion paper, Mihailov (2005), summarizes the developments in the British *institutional framework* for monetary policy making since the early 1990s and the current goals and instruments of the Bank of England. It relies only on *forward*-looking Taylor rules estimated via GMM and focuses on the *final* vs *real-time* data opposition to identify and interpret the *feedback* and *stance* of monetary policy in the UK. We here go beyond all of these aspects, addressing the specific question of the title in terms of robust analysis of Bank of England's reaction function.

where  $i_t^T$  is the current-period *target* nominal interest rate (NIR),<sup>3</sup>  $\bar{i}$  is the (constant) '*equilibrium*' NIR,  $p_t$  is (the log of) some general price index (which the monetary authority aims to stabilize) and  $\phi > 0$  is a response coefficient; or, alternatively,

$$\Delta i_t^T = \phi \pi_t,$$

where  $\pi_t \equiv \Delta p_t$  is the inflation rate over a preceding period of relevance to the corresponding (ex post) NIR.<sup>4</sup> The principal benefit from such a rule is that, as Woodford (2003) argues in chapter 2 of his book, it is able to stabilize the price index around a constant level.

**2.1.2. Monetary Rules and (In)Determinacy of Rational Expectations Equilibrium.** Research on monetary policy rules revived with a new impetus in the mid-1970s, following the rational expectations revolution in economic theory. Sargent and Wallace (1975), in particular, criticized *interest rate* rules of the Wicksellian (or Keynesian) kind from the point of view of the determinacy of rational expectations equilibrium, in the sense of a unique equilibrium satisfying certain bounds. They argued that, unlike *money supply* rules in the monetarist tradition, interest rate rules led to indeterminacy. McCallum (1981), however, made clear that this indeterminacy result applies only when the interest rate rule is specified as a function of *exogenous* variables, e.g., the history of exogenous disturbances. By contrast, if the nominal interest rate is specified as a function of *endogenous* variables – such as, for example, inflation in the Wicksellian rule above and the output gap in the Taylor rule to be discussed below – the problem of indeterminacy does not arise.

Further debates on monetary policy rules have materialized in proposals for alternative specifications. We briefly review the most important among them, which will be of relevance for clarity of exposition and for the econometric methods applied in the present study.

**2.1.3. The Goodhart Rule.** Similarly to Wicksell (1898, 1907) and again in a discursive style, Goodhart (1992) has more recently suggested a monetary policy reaction function that would algebraically – according to Woodford (2003), p. 40 – look like:

$$i_t^T = 3 + 1.5\pi_t.$$

Goodhart (1992), p. 324, interprets this rule in the sense that, starting from *zero* inflation and 3 per cent *nominal* interest rate, the *target* interest rate should rise by 1.5 per cent for each 1 per cent increase of inflation. Notice that the Goodhart rule has the *level* of the nominal interest rate set in proportion to *inflation*, with a coefficient of proportionality of 1.5. In the Wicksellian rule, by contrast, the relation is either between pure *levels*, as in the first specification (with  $i_t^T$  and  $p_t$  involved), or between pure *changes* (with  $\Delta i_t^T$  and  $\Delta p_t \equiv \pi_t$ ).

**2.2. The Taylor Rule.** The Taylor rule is an interest rate rule that also includes an *output gap* term in addition to an inflation (and *not* price level) term in the policy reaction functions implicitly suggested by Wicksell (1898, 1907) and Goodhart (1992). As Taylor (1993) has insisted and as Woodford (2003), p. 39, has stressed, such a feedback rule can be regarded both as a rough *positive description* of the way monetary policy had actually been made and as a straightforward *normative prescription* of how monetary policy should optimally be conducted.<sup>5</sup>

There is some ambiguity in the subsequent literature, in particular with respect to the alternative writings available in it for the *original* Taylor rule. We next clarify this – perhaps minor – detail, as we shall use the original Taylor rule as a point of departure and as a benchmark for comparison in both the theoretical and empirical part of the present study.

<sup>3</sup>Woodford (2003) uses the simpler notation  $i_t$ , but we would here and further down be explicit in distinguishing the *targeted* interest rate,  $i_t^T$ , from the *actual* one,  $i_t$ .

<sup>4</sup>More precisely,  $\pi_t \equiv \Delta p_t \equiv p_t - p_{t-k} \equiv \ln P_t - \ln P_{t-k} \equiv \Delta \ln P_t \equiv \ln \frac{P_t}{P_{t-k}}$  and  $\Delta i_t^T \equiv i_t^T - i_{t-k}^T = \phi(p_t - p_{t-k})$ , with  $k$  being an integer, most frequently 1 (for annual data), 4 (for quarterly data) or 12 (for monthly data); the rule written in *differences* of the interest rate thus results from the one in *levels*.

<sup>5</sup>The normative implication has emerged from stochastic simulation of a number of econometric models Taylor (1993) and Henderson and McKibbin (1993) undertook and published at nearly the same time.

2.2.1. *The Original Taylor Rule.* The monetary policy rule Taylor (1993) proposed was, in its original notation (without time subscripts), p. 202, eq. (1):

$$(2.1) \quad r = p + 0.5y + 0.5(p - 2) + 2,$$

where, in the words of Taylor,  $r$  is the federal funds rate,<sup>6</sup>  $p$  is the rate of inflation over the previous four quarters<sup>7</sup> and  $y$  is the percent deviation of real GDP from a target, i.e.,

$$y = 100 \times \frac{Y - Y^*}{Y^*},$$

with  $Y$  denoting real GDP and  $Y^*$  (linear-)trend real GDP, the latter growing by 2.2% per year for the Taylor (1993) sample, 1984:1 through 1992:2 (34 quarters).

The first constant 2 in (2.1), in the brackets, stands for the inflation target of the monetary authority and is assumed to be, in the US case for the sample period, 2% p.a.; the second constant 2, the last term in (2.1), is the ‘equilibrium’ real rate,<sup>8</sup> itself chosen so as to be close to the assumed *steady-state* growth rate of the economy of 2.2% (that is, as measured by the linear-trend real GDP growth). The interpretation Taylor (1993) himself, p. 202, suggested to (2.1) was the following:

”The policy rule ... has the feature that the federal funds rate rises if inflation increases above a target of 2 percent or if real GDP rises above trend GDP. If both the inflation rate and real GDP are on target, then the federal funds rate would equal 4 percent, or 2 percent in real terms.”

Taylor (1993) further emphasized that the policy rule he proposed had the *same* coefficient, 0.5 in (2.1), on the *deviation* of real GDP from trend *and* on the *deviation* of the inflation rate from target. This particular *numerical* relation has later been named the ‘Taylor principle’; yet it has often been interpreted in an equivalent, but differently written way: namely that the corresponding coefficient on inflation (and on its *deviation* from target too, in still another writing, as made clear below) should be 1.5. The essential point here is that the response of the monetary authorities to a rise in the inflation rate should be stronger, or elastic, in the sense of exceeding 1. Moreover, such a response has been claimed consistent with ensuring a unique, stationary, rational expectations equilibrium in the economy.

To see the link between the 0.5 coefficient on the deviation of inflation from target in the original Taylor rule and the 1.5 coefficient on inflation in subsequent interpretations of the rule, start by rewriting (2.1) as:

$$r = p + 0.5y + 0.5p - 0.5 \times 2 + 2,$$

$$r = \underbrace{(2 - 0.5 \times 2)}_{const} + 1.5p + 0.5y.$$

Then change the original notation for the *variables* in Taylor (1993) with a corresponding more standard and time-indexed notation,<sup>9</sup>  $r = i_t^T$ ,  $y = x_t \equiv y_t - y_t^P$  (with  $y_t^P$  denoting the *trend* in real GDP, often considered as the *potential* output, itself a natural GDP *target* for any government), and  $p = \pi_t$ , to obtain:

$$i_t^T = \underbrace{(2 - 0.5 \times 2)}_{const} + 1.5\pi_t + 0.5(y_t - y_t^P).$$

Now substitute the original numerically-expressed *constants* by respective general symbols – that is, the equilibrium real rate, 2, by  $r^*$  and the inflation target, 2, by  $\pi^T$  – to get:

$$(2.2) \quad i_t^T = \underbrace{(r^* - 0.5 \times \pi^T)}_{const} + 1.5\pi_t + 0.5(y_t - y_t^P).$$

<sup>6</sup>That is, the short-term interest rate target for monetary policy in the US context.

<sup>7</sup>In % p.a., that is,  $p \equiv 100 \times \frac{P_t - P_{t-4}}{P_{t-4}}$ , as implicit in Taylor (1993).

<sup>8</sup>Of *return* or of *interest*, Taylor (1993) remains not specific about that, although the real interest rate is rather understood in this particular context.

<sup>9</sup>In fact, our notation further down basically follows Walsh (2003), chapter 11, with also some influence from Woodford (2003), chapter 1. However, we adapt this notation a good deal, to suitably fit the purposes of our exposition and the techniques applied later.

(2.2) is the original Taylor rule reformulated in terms of the actually observed current-period rate of inflation,  $\pi_t$ . One can now see why the coefficient on *actual* inflation implied by the originally specified Taylor rule should be 1.5, exactly the *same* as the analogous one in the Goodhart rule above!

Further, add and subtract  $1.5\pi^T$  in the right-hand side (RHS) of (2.2):

$$i_t^T = r^* - 0.5 \times \pi^T + 1.5\pi^T + 1.5\pi_t - 1.5\pi^T + 0.5(y_t - y_t^P),$$

to finally arrive at

$$(2.3) \quad i_t^T = \underbrace{(r^* + \pi^T)}_{\equiv i^T = \text{const}} + 1.5(\pi_t - \pi^T) + 0.5(y_t - y_t^P).$$

(2.3) is the original Taylor rule reformulated in terms of the (constant) *nominal* interest rate (NIR) target,  $i^T \equiv r^* + \pi^T$ , implied by a situation in which the economy has achieved both other targets, for inflation and for output, so that both deviation terms in (2.3) are zero. As clear from the definition, the targeted (constant) NIR equals the equilibrium (constant) *real* interest rate (RIR),  $r^*$ , plus the (constant) target rate of inflation,  $\pi^T$ . The latter inflation target is thus, in effect, assumed to be the anchor to which expected inflation would converge. One can also easily see from (2.3) why the coefficient on the *deviation of actual* inflation from *target*,  $\pi_t - \pi^T$ , often termed *inflation gap*, should be 1.5 as well, in the version of the original Taylor rule with intercept equal to the equilibrium RIR plus the inflation target.

Note, however, the differences across (2.1), (2.2) and (2.3). (2.1) suggests *numerically identical* equilibrium RIR, potential (or trend) output, and inflation target, all (constant and) equal to 2% p.a. ( $r^* = y^P = \pi^T = 2$ ) and – in a symmetric way – *quantitatively identical* policy response coefficients (0.5 each) on the deviation of both actual inflation from target and actual output from potential. (2.2) is, as we said, a reformulation of (2.1) in terms of *actual* inflation, allowing econometric estimation when the inflation target is unknown; in it, the response coefficient to inflation duly changes to 1.5, with the one for the output gap remaining 0.5. (2.3), finally, is a third version of the original Taylor rule, allowing for an interpretation of the intercept of the regression as the *desired* (constant) NIR in equilibrium,  $i^T \equiv r^* + \pi^T$ ; the policy reaction parameters are again quantified at 1.5 and 0.5, but now with regard to the *deviation* of inflation and output from their respective targets (as in (2.1) but now with a different intercept). We point out to the particular *numerical* values of the response coefficients in the feedback rules originally proposed by Taylor (1993) and Goodhart (1992) because their (relative) magnitude has often been discussed in the literature, usually without the distinction among the three versions we introduced here having been made clear or well understood. We shall also refer to these quantitative benchmarks for the policy response parameters when discussing later on our related results.

**2.2.2. Classic Taylor Rules.** Generalizing the policy rule still further – and adapting again largely Walsh's (2003) textbook notation, as we mostly did above<sup>10</sup> – we could replace the *numerical* coefficients which Taylor (1993) postulated (without attempting to estimate at all) as "round numbers that make for easy discussion" (p. 202) by corresponding letters denoting *any* constants, i.e., parameters that can be estimated from the data:

$$(2.4) \quad i_t^T = i^T + b_{\pi,0}(\pi_t - \pi^T) + b_{x,0}x_t,$$

with  $i^T \equiv r^* + \pi^T$ ,  $x_t \equiv y_t - y_t^P$ , and  $b_{j,l}$  denoting the coefficient to the respective *variable* of interest (expressed by the relevant *letter* according to our notation)  $j = \pi, x, i$  ( $j = 0$  stands for some compound intercept terms later on, as will become clear) at a respective *lag*(-)/*lead*(+) (expressed by an integer *number*)  $l = \dots, -2, -1, 0, +1, +2, \dots$  ( $l = 0$  designates, of course, a contemporaneous, or current-period, response). We can estimate (2.4), as specified with contemporaneous response parameters, directly from the data (for  $i_t^T$ ,  $\pi_t$  and  $x_t$ ) if we *know* the inflation target ( $\pi^T$ ); if we do *not* know it, (2.4) can be written as

<sup>10</sup>Except that Walsh (2003), chapter 11, uses  $x_t$  to designate the *gap* in real GDP. We shall switch to the  $x_t$  output gap symbol as well from now on. Yet up to here we have preferred to keep our notation  $y_t - y_t^P$  because it makes *explicit* that the variable in question is the real GDP *gap*: this is important, especially in the case of Taylor rule empirical estimation, since some authors have argued against the use of an imprecise and methodologically contentious output gap measure and in favour of a version of the rule where real GDP *growth* replaces the real GDP gap. We present further down econometric estimates of both these types of policy reaction functions. Notice as well that  $y_t - y_t^P$  can be reduced to  $y_t - y^P$ , as implied, in fact, by the original Taylor rule. In an inverse sense,  $\pi_t - \pi^T$ , as above, can be left more general by writing it as  $\pi_t - \pi_t^T$ , so that an inflation target which changes through time may also be allowed for.

$$(2.5) \quad i_t^T = \underbrace{(i_t^T - b_{\pi,0}\pi^T)}_{\equiv b_{0,0}=const} + b_{\pi,0}\pi_t + b_{x,0}x_t$$

and estimated in the form of (2.5). Observe at this point a tricky interrelation (or transformation) across the *intercept* terms in the above Taylor rule equations, usually not very explicit in the literature:

$$(2.6) \quad b_{0,0} \equiv i^T - b_{\pi,0}\pi^T \equiv \underbrace{r^* + \pi^T}_{\equiv i^T} - b_{\pi,0}\pi^T \equiv r^* + (1 - b_{\pi,0})\pi^T = const,$$

so that  $b_{0,0} \neq i^T \neq r^* \neq \pi^T \neq b_{\pi,0}$ .

Note as well another analytical insight from our detailed discussion here, which often remains ambiguous (even if implicit) in the numerous and diverse papers on Taylor rules: estimating (2.4) when the target rate of inflation is *known (and constant)* will result in arriving at the *same* policy response parameters (in sign and magnitude) as estimating (2.5) when the inflation target is *unknown (but constant)*,  $b_{\pi,0}$  in both equations (and also  $b_{x,0}$ ). In such parallel regressions, the only difference across the two estimation specifications will be the magnitude (and, potentially, sign) of the intercept:  $i^T$  in (2.4) and  $b_{0,0}$  in (2.5).

(2.4) with  $\pi^T = 2.5\%$  p.a. (being most appropriate for the UK during the *inflation targeting* period, as discussed further down) and (2.5) were, in fact, the first Taylor rule equations we estimated, by ordinary least squares (OLS) and by two-stage least squares (TSLS), as we report later on. Following in part Woodford (2003), p. 21, we would refer to this more general version of the original Taylor rule, insofar all of its coefficients are not restricted to specific numerical values, as *classic* Taylor rules.

The subsequent literature has criticized the original (or classic) Taylor rule, mostly because of the serial correlation usually found in the error term and the potential endogeneity problem arising from the contemporaneous regressors in it, the latter also posing unrealistic informational requirements to the monetary authority. Feedback functions of the type have therefore been augmented in a number of ways. We summarize below the most important among them, also corresponding to the versions we estimated on UK data.

**2.2.3. Naïve Taylor Rules.** To address some of the critiques towards the original Taylor rule, in particular the problem of endogeneity, research in the area has further shifted to estimating what Nelson (2000), p. 13, calls *naïve* Taylor rules. In a *naïve* Taylor rule both gap measures, in real GDP and in inflation, are included with *one lag*, thus precluding the possibility for a correlation between the explanatory variables and the disturbance term when running regressions. In addition to that econometric justification, an informational one has been added: it is not realistic for the central bank to dispose of (precise) *current-period* information (although *forecasts* can be used) when reacting to changes in the economic and monetary environment, so that the contemporaneous response to the regressors in the classic Taylor rule has thus been eliminated as well. The reason for branding this type of feedback rules ‘naïve’ is, according to Nelson (2000), to distinguish it from reaction functions which incorporate *more extensive dynamics*, in particular interest rate *smoothing* or *forward-looking* policy behavior, as we shall discuss in greater detail below.

To be more precise, Nelson (2000) specifies and estimates by OLS naïve Taylor rules of the form:

$$(2.7) \quad i_t^T = \underbrace{(i_t^T - b_{\pi,-1}\pi_{t-1}^T)}_{\equiv b_{0,-1}=const} + b_{\pi,-1}\pi_{t-1} + b_{x,-1}x_{t-1},$$

where *actual* inflation registered during the *past* period replaces the *current-period deviation* of inflation from target in the original Taylor rule and where the output gap enters as well with a one-period lag. Notice that  $b_{0,0}$  in (2.5) is not the same as  $b_{0,-1}$  in (2.7) insofar the latter regression is estimated with a *lag* of 1 relative to the former.

**2.3. Generalized Taylor(-Type) Rules.** Continuing widespread problems of residual autocorrelation in econometric estimation of classic and naïve Taylor rules have further led to two approaches. One of them has tried to identify the order of serial correlation and duly correct for it, by adding *autoregressive error terms* in the estimated regressions (mostly of order 1, with quarterly data). The second common approach has been to introduce a more realistic partial adjustment model for the nominal interest rate,



effectively resulting in including a *lagged dependent variable*. The latter type of policy reaction functions has become known as interest rate smoothing Taylor rules.

2.3.1. *Interest Rate Smoothing Taylor Rules.* Central banks usually implement their policy in a way that smooths their operating instrument, i.e., the *short-term nominal interest rate*.<sup>11</sup> This is consistent with a Taylor rule specification of the form<sup>12</sup>

$$(2.8) \quad i_t^T = b_{0,0} + b_{\pi,0}\pi_t + b_{x,0}x_t + b_{i,-1}i_{t-1}.$$

We return to such *interest rate smoothing* policy reaction functions and their microfoundations when discussing the New Keynesian theoretical underpinnings of our GMM estimation in section 4.

2.3.2. *Backward-Looking (or History-Dependent) Taylor Rules.* Combining (2.7) and (2.8) and allowing for a richer dynamics has then led to *backward-looking* reaction functions, which can generally be written in the form:

$$(2.9) \quad i_t^T = b_{0,-n} + \sum_{n=1}^N b_{\pi,-n}\pi_{t-n} + \sum_{n=1}^N b_{x,-n}x_{t-n} + \sum_{n=1}^N b_{i,-n}i_{t-n},$$

with the dynamic structure truncated at some relevant lag length  $N$ . This is the backward-looking rule which Nelson (2000, 2001, 2003) estimated for  $N = 1, 2$  across 5 sample periods of British quarterly (and monthly, for the shortest subsample) data between 1972 and 1997 in his ‘Taylor rule guide to UK monetary policy’. Note that it is again simply the *actual* inflation rate, not its *deviation* from target, which enters (2.9). Furthermore, Nelson (2000, 2001, 2003) restricts all three RHS variables to have the *same* lag length,  $N$ , which is too stringent. In general, the lag length can be different for each of the explanatory variables and selected according to a relevant statistic such as the Akaike or Schwartz information criteria. However, estimation of all sorts of *backward-looking* Taylor rules is nowadays oversimplistic, given that rational agents, including the central bank, anticipate and forecast the variables of key interest to them. We therefore emphasize our results from *forward-looking* rules for the UK, although we estimated backward-looking rules as well. Nelson (2000, 2001, 2003) applied a similar approach but for periods *before* operational independence and relying on OLS and TSLS, whereas we have also used GMM over our two inflation targeting subsamples – notably, over the one *after* operational independence – as will be explained in more detail in section 4.

2.3.3. *Forward-Looking (or Rational-Expectations) Taylor Rules.* In accordance with considerations of rationality in economic behavior, the literature on policy reaction functions has further developed to complement backward-looking specifications with *forward-looking* ones. Such versions of the Taylor rule assume rational expectations, not naïve extrapolation. Moreover, *forward-looking* central bank feedback functions have recently been grounded in microfounded macroeconomic theory, and thus have a deep theoretical justification.

2.3.4. *Optimal Policy Rules: Instrument Rules vs Targeting Rules.* Woodford (2003), p. 58, has recently argued that *optimal* policy rules, i.e., feedback rules derived from explicit models, such as the New Keynesian model of monetary policy (without and with microfoundations) will usually be both forward-looking and backward-looking, and that the lead and lag horizons in them will not be too long.

”... Moreover, the optimal rules that I obtain are also typically different in their dynamic specifications. Optimal rules are history dependent in ways other than those of the classic Taylor rule or familiar descriptions of inflation-forecast targeting; and while they may well be more forward-looking than the classic Taylor rule, in all of the calibrated examples they are considerably less forward-looking than the procedures currently used in the inflation-targeting central banks.” Woodford (2003), p. 58.

<sup>11</sup>More recently in most advanced market economies, the overnight *repo rate* in the market for bank reserves (also more generally known as the money market, hence the *money market rate*).

<sup>12</sup>We should have, more precisely, written  $i_t^T = b_{0,0}^{sm} + b_{\pi,0}\pi_t + b_{x,0}x_t + b_{i,-1}i_{t-1}$  instead, where the *sm* superscript to  $b_{0,0}$  distinguishes this particular intercept from earlier ones on the basis that now an *additional*, lagged dependent variable is included in the regression. However, we have decided to avoid any further complication of the notation when accounting for details of a lesser importance. Moreover,  $b_{0,0}$  in (2.5) =  $b_{0,0}$  in (2.8), apart from the added third regressor.

That is why we focus in the analytical as well as empirical part of our study on Taylor rules that are both forward-looking, in inflation and the output gap, and backward-looking, in the interest rate, but at the same time have a simple(r) dynamic structure.

Svensson (1999, 2003) insisted to distinguish *targeting* rules as opposed to *interest rate* rules of the kind described thus far. In fact, targeting rules are a major competitor of Taylor rules in the recent literature as well as practice on monetary policy. Perhaps the best-known example of a targeting rule is *inflation-forecast targeting*. Vickers (1998), among others, has used it to explain the monetary policy framework implemented since 1992 in the United Kingdom, which is of particular relevance to the present study. There is no formula prescribed for setting an *interest rate operating target* under inflation-forecast targeting. The Bank of England is free to set this target at whatever level is consistent with its inflation forecast in order to meet a certain *target criterion*. The latter criterion may however well resemble, as Svensson (1999) has pointed out, the right-hand side of a forward-looking Taylor rule (without the interest rate smoothing term).

Woodford (2003), e.g., p. 58, furthermore argues that *optimal* rules can easily take the form of *generalized* Taylor rules or of *target* criteria for a forecast-targeting procedure. We have therefore not discarded the Taylor rule as a potential ex post description of actually materialized monetary policy only because, on a normative basis, the Bank of England does not explicitly follow an instrument rule but a targeting rule. We believe that our results support to a large extent the usefulness of Taylor rules in deriving certain lessons on the actual outcomes of central bank behavior, even in the UK under inflation targeting. As a first study of the particular issue of interest here, our paper restricts attention to rather conventional functional forms and estimation approaches. Further research could, of course, go into more detail and complication.

**2.3.5. Real GDP Growth Instead of Real GDP Gap.** Mostly because of the well-known problems in measuring the *true* output gap, which cannot be observed, some researchers have proposed to use the rate of real GDP *growth* instead of the real GDP *gap* when estimating central bank policy reaction functions. Such feedback rules, however, seem to us inappropriate for this purpose, essentially because of theoretical reasons. We shall try to make the point clearly later on, while discussing our principal findings.

**2.3.6. Exchange-Rate Augmented Taylor Rules.** The literature on Taylor rules has gone further to estimate specifications that *explicitly* include one or more (contemporaneous and lagged) exchange rate terms. This has been found logical especially in the case of small open economies. However, Taylor (2000) argues that there is no need to do so, since even if the exchange rate may matter a lot for a small open economy, its dynamics will be reflected (almost immediately) in the dynamics of the price level, that is, in inflation as well. So, once an inflation term is included in the Taylor rule, the exchange rate is always *implicit* in the equation, via its pass-through onto consumer prices. We have, nevertheless, controlled for an exchange rate term, and our results will be reported in section 4.

**2.3.7. Nonstationary Taylor-Type Policy Rules.** As recently pointed out by Gerlach-Kristen (2003), the empirical literature on policy reaction functions has usually ignored the issue of (*non*-)stationarity of the variables taken into account. She explores the econometric properties of the traditional Taylor rule model using euro area quarterly data for 1998-2002 and finds signs of instability and misspecification. She then estimates interest rates rules using the *cointegration* approach and claims that such rules are stable in sample and forecast better out of sample. The findings of Gerlach-Kristen (2003) are certainly of interest. Moreover, nonstationarity seems relevant for part – but not all – of the UK time series we include in our Taylor rule estimation, as we discuss later on. In this sense, a cointegrated approach may deserve attention in future research. In our case, however, because of (i) the relatively short subsamples (the pre-independence one containing 18 quarterly observations) and (ii) the low power of unit root tests in such short (sub)samples, as well as, more importantly, because of (iii) the theoretical reasons for stationary rates of interest and inflation and for a stationary output gap and (iv) the likely stationarity of our Hodrick-Prescott filtered measure for the real GDP gap, we keep in the tradition – defended notably by Clarida-Gali-Gertler (1997, 1998 a, b, 2000) – and abstain from cointegrated reaction functions in this particular study.

**2.3.8. Nonlinear Taylor-Type Policy Rules.** More recently, the literature has also turned to explore potential *nonlinearities* in the feedback rule. For example, Martin and Milas (2004) and Kesriyeli, Osborn and Sensier (2004) have directly addressed such issues with UK data and Surico (2004) with US data. No matter that this is another interesting, and perhaps promising, avenue for future research, in this

first attempt to learn something about the effects of the Bank of England's operational independence on inflation targeting by examining Taylor rules we here abstract from nonlinear functional forms. Nevertheless, we do find evidence for another kind of nonlinearity, in the sense of an *asymmetric* policy response of the UK monetary authority before and after operational independence to the output gap, largely due to the business cycle, as we argue further down.

**2.3.9. Hybrid Monetary Policy Rules.** A final notion to mention in the taxonomy of monetary policy reaction functions we briefly reviewed in the present section is the *hybrid* rule. These are rules which include *both inflation* and the *price level* as policy response variables, in addition to the *output gap*. As Jääskelä (2005) points out, the debate on price level and inflation targeting was triggered by Fischer (1994), and a substantial literature developed out of it in the last decade. Nessén and Vestin (2000), for instance, show that the performance of a hybrid target can be superior to a price level target and to an inflation target, taken separately, if commitments of an inflation targeting central bank are not feasible. Batini and Yates (2003), on the other hand, study the pros and cons of (*non-optimized*) hybrid rules in an open-economy context when policy makers are able to commit. A conclusion in Jääskelä (2005) is that it does not make sense to include the price level in a policy rule when inflation expectations are backward-looking; but when they are forward-looking, the price level rule and the hybrid rule are superior to the standard (inflation-based) Taylor rule under *certainty* about the *structural* parameters of the model; however, the standard (*optimized*) Taylor rule is *more robust* to *model uncertainty* than both those alternatives. This is another reason to focus our initial analysis here on the simplest case of forward-looking Taylor rules with interest rate smoothing, rather than hybrid, nonlinear or nonstationary ones, before potentially extending it in ways to incorporate the more complex features discussed in the last few paragraphs.

### 3. DATA AND PRELIMINARY TESTS

#### 3.1. Data.

**3.1.1. Sources and Frequency.** We employ standard time series that are common in Taylor rule estimation. However, we also make use of a few *alternative proxies* for the explanatory variables, which are of particular relevance for the UK.

All data were downloaded from the statistical pages on the websites of the UK Office of National Statistics (ONS) and the Bank of England (BoE).

As mentioned, we work here with *quarterly* frequency. This certainly makes our subsamples smaller than if we had recurred to monthly time series. Yet our quarterly estimates turned out most of the time to be both significant in econometric terms and interpretable in economic terms. This is in part because, as Clarida-Gali-Gertler (1997, 1998 a, b, 2000) and Nelson (2000, 2001, 2003) have pointed out with respect to their earlier and similar estimations, the variability of the data involved is sufficient to produce reasonable results even in relatively small samples.

#### 3.1.2. Variable Proxies.

**Nominal Short-Term Interest Rate.** Following most other previous Taylor rule papers on the UK, in particular Nelson (2000, 2001, 2003) and Martin and Milas (2004), we assume that the short-term interest rate supposed to be the operating instrument of the Bank of England is best proxied by the *3-month Treasury bill rate*. This is not quite precise, because since operational independence the Bank has been using the *2-week repo rate* as its policy instrument. Yet the latter rate has been relatively recently introduced, i.e., in May 1997. As Nelson (2000, 2001, 2003) points out, the advantage of the 3-month Treasury bill rate is that, being very close to the various different rates – four in total since the early 1970s – that have played the role of operating instrument,<sup>13</sup> it can be used, for greater comparability and with no much loss of precision, to approximate all of them when longer periods of study are of interest (see figures 1 and 2 in Appendix A).

**Inflation.** Inflation is proxied in our study by two alternative indexes that are usual choices when working with UK data:

- the *Retail Price Index* (RPI), as in Martin and Milas (2004) and Kesriyeli, Osborn and Sensier (2004), among others; and

<sup>13</sup>Including also – in addition to the currently used *repo rate* – the *bank rate* (through September 1972), the *minimum lending rate* (October 1972 - July 1981) and the *minimum band 1 dealing rate* (August 1981 - April 1997).

- the same Retail Price Index but *eXcluding* the *mortgage rate* (RPIX),<sup>14</sup> as, for instance, in Nelson (2000, 2001, 2003).

The RPIX has been the officially announced measure of UK inflation and guide for UK monetary policy in the period 1992-2003, and the RPI has performed that same role before 1992 (see Figure 3, which documents the high correlation between these two price level indexes, and Figure 5, which presents the two respective inflation measures before and after operational independence). As for the consumer price index (CPI), which is the standard measure of inflation in most other economies, including for the purposes of monetary policy, it has become the official index accounting for the evolution of the UK general price level only since 2004, and has in this way precluded any possibility to use it in our study. Output Gap. Our measure for the output gap is, alternatively, constructed out of two available time series:

- the *final* or *revised* data for real GDP, as in the majority of studies on Taylor rules; and
- the *real-time* or *initially released* data for the same variable, real GDP, which have been available to policy makers ‘in real time’, that is, at the time of making decisions on monetary policy: more precisely, we use the series constructed by Nelson and Nikolov (2001) and accessible on the Bank of England’s website (see Figure 4, which documents another high correlation, this time between our two UK output level proxies, as well as Figure 6, which compares their growth rates before and after operational independence). In a series of papers, Orphanides (1998, 2000, 2001, 2003) first argued that real-time data, in addition to being more realistic, may overturn some conclusions about feedback rules based on final data.

Moreover, each of these two types of real GDP series has been filtered by two now standard (although not perfect) procedures to obtain a measure for the output gap, namely:

- by fitting a *quadratic* trend, as in Clarida, Galí and Gertler (1997, 1998 a, b, 2000) and Nelson (2000, 2001, 2003), among others; and
- by a *Hodrick-Prescott* detrending (with a smoothing parameter equal to 1600, as recommended for quarterly data), as in Martin and Milas (2004) and Kesriyeli, Osborn and Sensier (2004), among others.

Both of these methods to obtain an output gap measure have, of course, their advantages and shortcomings. For this reason, and also to arrive at results that are not necessarily sensitive to the detrending employed, we have preferred to work with both filtering procedures, as reported further down (see figures 7 and 8).

**3.1.3. Graphs and Descriptive Statistics.** *Graphs* – figures 1 through 8 – illustrating the dynamics of our data as well as *tables* with descriptive statistics for the two periods of interest in the present study, *pre*-independence inflation targeting (1992:4-1997:1) in Panel A of Table 1 and *post*-independence inflation targeting (1997:3-2004:4) in Panel B of Table 1, are included in Appendix A.

## 3.2. Preliminary Tests.

**3.2.1. Seasonality Tests.** Information contained in the files downloaded from the sources of our data, the ONS and the BoE, indicated certain inconsistency of the time series we wished to employ in the Taylor rule estimations with respect to their seasonal adjustment. More precisely, nominal GDP data and the GDP deflator – hence, real GDP, by construction – were provided at their source as *seasonally adjusted* (sa), whereas both price levels, the RPI and the RPIX, as well as the 3-month Treasury bill rate were *not seasonally adjusted* (nsa).

We thus performed Census X12 seasonality tests with all these series. As was expected, the real GDP data were not found to display seasonality while for the price level and interest rate data it was definitely confirmed that they followed a seasonal pattern.<sup>15</sup> Consequently, we performed *two* versions of our Taylor rule regressions:

- with the *raw* data, as they were from their sources, i.e., with no seasonal adjustment to the RPI, the RPIX and the 3-month Treasury bill rate; and
- with *seasonally adjusted* – by the Census X12 procedure – respective price levels (denoted in our tables in Appendix A as RPISA and RPIXSA) and interest rate.

<sup>14</sup>The main reason for this exclusion has been claimed to be that the mortgage rate tends to move closely with the Bank of England’s instrument.

<sup>15</sup>Detailed results from our *seasonality* tests are available upon request.

We did so because of certain criticisms in the sense that deseasonalization techniques may diminish or eliminate important features of the raw time series and thus give rise to findings which do not necessarily reflect genuine correlations across the data. On the other hand, it seemed to us somewhat inconsistent not to employ seasonally adjusted prices and interest rates side by side with real GDP data that were anyway seasonally adjusted at the source. This is why we later on report both type of results (nsa and sa).

**3.2.2. Stationarity Tests.** Another typical preliminary procedure in time series analysis is to test for (non)stationarity of the variables entering the regressions. By the way, in the particular case of Taylor rule estimation, this has not been systematically done in most of the previous literature, as Gerlach-Kristen (2003) has duly pointed out.

We followed her critique and tested our variables for (non)stationarity, applying three standard unit root tests, each one in four alternative specifications. More precisely, we performed Augmented Dickey-Fuller (ADF) unit root tests based on autoregressive models in parallel with kernel-based Phillips-Perron (PP) unit root tests, with the null for both tests being that of a unit root (i.e., nonstationarity) present. These two tests were further supplemented by a test constructed on the *opposite* null, of stationarity, namely the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test, and both autoregressive and kernel-based specifications of it were used.<sup>16</sup>

We generally found that the price levels, RPI and RPIX, could be either  $I(1)$  or  $I(2)$ . Hence, inflation could be either stationary or not, depending on the chosen proxy and test. The 3-month Treasury bill rate and the real GDP gap obtained from quadratic-trend fitting cannot be treated with certainty neither as definitely stationary nor as definitely  $I(1)$  variables either, because of mixed findings from the alternative unit root tests and specifications within each test we resorted to. Only the real GDP gap obtained from Hodrick-Prescott detrending appeared to be most likely  $I(0)$ . Having found no overwhelming evidence that the variables which – according to the theory – should enter our Taylor rule equations were integrated of the *same* order, we thus also avoided any idea for estimation based on the cointegration approach, as recently done in a similar study for the euro area by Gerlach-Kristen (2003).

With (i) such a heterogeneity in the likely order of integration of our time series and (ii) bearing in mind as well the notorious low power of unit root tests, in particular in short samples like ours, we ultimately followed the New Keynesian theory of monetary policy and performed Taylor rule estimation in the standard way, as also argued and done recently by Clarida, Galí and Gertler (1997, 1998 a, b, 2000). These authors defend the key assumptions in their work – stationarity of inflation and the nominal interest rate, as we shall also assume here – by stressing that they are both empirically and theoretically plausible.

What we could do, and what we did, in fact, to at least partly address or mitigate the problem in question and hopefully obtain somewhat better results, of a greater robustness and of a higher generality, is that we used three alternative methods of estimation and a number of econometric specifications within each method. We then looked for results that are generalizable to a high extent, i.e., such that hold across most of the performed regressions. We discuss in the next section our estimation strategies and specification choices, by also linking them to the previous literature and motivating our preferred approaches.

#### 4. ESTIMATION METHODS, SPECIFICATIONS ESTIMATED AND KEY FINDINGS

Our overall empirical strategy was to apply the most common and appropriate techniques used by now in similar Taylor rule studies. These techniques relate to ordinary least squares (OLS), in the earliest literature, and to two-stage least squares (TSLS) and the generalized method of moments (GMM), in the more recent papers. Another objective we pursued was to begin from the simple and most natural specifications, including the original rule by Taylor (1993), and move to the more complicated Taylor rule versions and econometrically better suited and justified techniques, thus basically following the chronology in which the literature evolved. We therefore started with a logical point of departure, by estimating the original Taylor rule (in a few versions). Tests for structural breaks were then performed on it as well, essentially to check the validity of our split of the sample in two subperiods. Yet for theoretical and econometric reasons we duly make clear further down, we focus on, and give most weight

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<sup>16</sup>Detailed results from our *stationarity* tests are available upon request.

to, our findings when subsequently employing the GMM approach to estimating forward-looking Taylor rules popularized by Clarida, Galí and Gertler (1997, 1998 a, b, 2000).<sup>17</sup>

We already said in the beginning that our paper differs from most preceding ones on similar issues in two aspects. First, our topic is *operational independence* of the Bank of England since mid-1997 and what its introduction has changed, if at all, in the perception – if not in the implementation itself – of monetary policy as recovered from the data, that is, by employing Taylor rules. To the best of our knowledge, the present study is the first to address this important question, of increased central bank independence, in an *inflation targeting* framework. Second, having resorted to a wide variety of econometric methods and regression specifications and having thus obtained a diversity of numerical measures on the key coefficients of interest – interpreted in the literature as the *reaction* of the monetary authorities to *inflation* and to the *output gap* and the *degree of interest rate smoothing* – we then look out for, and stress, certain generally valid results, i.e., those that survive across most methods and specifications implemented in terms of (i) *statistical significance*, (ii) theoretically correct *sign* and (iii) reasonable and economically interpretable (range of) *magnitude* of the above-enumerated policy response parameters. It is true that the greater the number of techniques and specifications an econometrician tries, the less is the chance for some general patterns to be uncovered in the ‘forest of numbers’. Nevertheless, we are able to identify results whose substance withstands all employed methods and regression variants and lends itself to a reasonable and insightful interpretation. The import of the present paper is to point out to a few such novel findings, on which we elaborate in more detail further down.

**4.1. Ordinary Least Squares: Classic and Backward-Looking Taylor Rules.** As a first benchmark – and to follow some sort of chronology or, rather, logic – we begin by reporting regression output from the straightforward, original Taylor rule and the simplest of the econometric methods applied, OLS. In what follows we then (i) extend the classic Taylor rule in several directions recommended in the literature and (ii) improve on estimation by adding results obtained through TLS and GMM, with a considerable diversity of alternative reaction function specifications and variable proxies. As noted, we place a predominant weight on our GMM results. Yet our *general* findings are not so method-specific, and can be – in part – detected even by less sophisticated feedback rules and econometric techniques. This is another reason to start by the simpler.

**4.1.1. Point of Departure: The Original (or Classic) Taylor Rule.** We estimated the Taylor (1993) rule on UK quarterly data for our two subsamples, in both its *original* specification (2.4), with  $\pi^T = 2.5$  as most appropriate for our particular country case and time period, and *classic* version (2.5), i.e., transformed *without* a constant inflation target *inside* the inflation gap variable in the RHS. We present our results in the first pair of columns in Table 2 in Appendix A. As pointed out in section 2, the only difference in the estimated coefficients is in the *intercept* term ( $i^T$  vs  $b_{0,0}$ ). This intercept could be more directly interpreted in the *original* Taylor rule, (2.4), in the sense of being the *desired* nominal interest rate,  $i^T$ . We find a statistically significant and economically reasonable magnitude for this NIR target in both subsamples: 5.76% (with Hodrick-Prescott detrending) to 5.77% (with quadratic detrending) p.a. during pre-independence and 4.95% (with quadratic detrending) to 5.01% (with Hodrick-Prescott detrending) p.a. during pre-independence.  $i^T$  is itself the sum of the (constant) ‘equilibrium’ real interest rate,  $r^*$ , and the (constant) inflation target,  $\pi^T = 2.5$ . Our measure of  $i^T$  thus implies, for  $\pi^T = 2.5$ , a corresponding measure of the equilibrium real rate of interest,  $r^* \equiv i^T - \pi^T$ : 3.25% p.a. pre-independence and 2.50% p.a. post-independence, both values making good economic sense with view to the British circumstances.

The first thing we would like to stress here is that the *original* Taylor rule performs in an impressive manner during our *post*-independence subsample. All variables (i) are statistically significant at all conventional levels and have (ii) the expected (from theory) sign and (iii) magnitudes that seem quite reasonable and completely interpretable. Moreover, the coefficients  $b_{\pi,0}$  and  $b_{x,0}$  are found practically the same, 0.85, as Taylor (1993) argued (however, he quantified them both at 0.5 instead). The only major problem with the regression results in Table 2 is *serial correlation* (reflected in the value of the

<sup>17</sup>Another recent estimation technique, which we do not pursue here, was implemented by Muscatelli, Tirelli and Trecroci (2000). They apply the *structural time series* (STS) approach proposed by Harvey (1989) to *generate* series of the *expected* inflation rate and output gap. By contrast, the Clarida-Galí-Gertler (1997, 1998 a, b, 2000) GMM approach essentially consists in using the *errors-in-variables* method to *model* rational expectations: in it, instead of forecasting inflation and output – e.g., by Kalman filter methods, as in Muscatelli-Tirelli-Trecroci (2000) – future *actual* values replace as regressors *expected* values, as we explain in more detail later on.

Durbin-Watson statistics). In the *pre*-independence subsample another problem seems to be that the *output gap* is statistically indistinguishable from zero, no matter which measure we use for it.

The second point we would like to make is that, however, this latter result does *not* seem too *surprising* in light of our findings – of a similar spirit although not exactly the same – when using more complicated Taylor rule specifications and more sophisticated econometric techniques, as reported further down. One interpretation may be in the sense that the Bank of England has not (systematically) considered the output gap in designing its monetary policy during 1992-1997 but has (consistently) reacted to it, as well as to inflation, during 1997-2004. Moreover, the coefficients on inflation do not unambiguously indicate that the response to it by the BoE has increased or decreased in magnitude in the post-independence period relative to the pre-independence one. We return with more analysis and a plausible interpretation to these initial findings in the later parts of the present section.

A third point is that, as is well-known from the related literature, Taylor rule equations in the original version often suffer from *serial correlation*. In our case, Lagrange multiplier Breusch-Godfrey tests, in addition to the Durbin-Watson one, have established a likely positive autocorrelation of the residuals of the regression of order 1. When an *AR*(1) correction in the error process is introduced in the equation we estimated, with no any other modification, our results do not change qualitatively (although in quantitative terms policy responses are twice weaker), as can be seen in the second pair of columns in Table 2.

4.1.2. *Structural Break Tests.* To check for structural breaks in (and, more generally, out of) our sample and, in essence, to confirm econometrically the choice of our sample split, quarter 2 in 1997, suggested by the narrative evidence reported below, we performed Chow *breakpoint* and *forecast* tests on the classic Taylor rule (2.5). The *dates* we selected for the tests are potentially the most likely ones to have resulted in structural instability in UK monetary policy throughout the 1990s and until 2004. All of these changes in monetary regime have been implemented following *official public announcements*, as also discussed by Nelson (2000, 2001, 2003), among others. These changes in policy can thus be considered as (largely) *exogenous*. The associated dates of their (approximate) perception as public knowledge, for which we have run the two mentioned Chow tests, are the following.

- (1) Membership of the British sterling in the *Exchange-Rate Mechanism* (ERM) of the European Community, as from October 1990.
- (2) Sterling crisis and suspension of the ERM in the UK, in September 1992, followed by the instauration of an *inflation(-forecast) targeting* framework for monetary policy as from October 1992.
- (3) Target inflation reformulated from a target *band* (or *range*) of 1% to 4% (implying a *mid*-point of 2.5% p.a.) to an explicit medium-term *point* target of 2.5% p.a. – as reported, for instance, in Martin and Milas (2004), p. 210, and, in more detail, in Haldane (1995) – but defined in an *asymmetrical* way (our next paragraph clarifies what this means).
- (4) The Bank of England granted *operational independence* from HM Treasury in May 1997, and in June 1997 the 2.5% point target announced to become *symmetrical*: i.e., to give *equal* weight to circumstances in which inflation is higher or lower than the target rate; in other words, inflation below the target was to be judged as being just as bad as inflation above the target.<sup>18</sup>
- (5) In December 2003, target inflation *lowered* from 2.5% p.a. to 2% p.a., and *expressed* as from January 2004 in terms of the Harmonized Index of Consumer Prices (HICP), renamed (again in December 2003) to simply the Consumer Price Index (CPI), instead of in terms of the RPIX (as it had been during the 1992-2003 period).

With view to this narrative account of the evolution of the framework for monetary policy in the UK provided by the Bank of England, a competent and credible source of primary information, we would argue that: (i) there has been no considerable change from the point when we break our sample in two subsamples, namely the *second quarter of 1997* (itself *excluded* from our estimation), when the Bank of England was granted *operational independence*, until (almost) the end of our sample; the only other major change of a similar magnitude is the break point, namely the *third quarter of 1992* (also *excluded* from our estimation), after which our sample starts with the introduction of *inflation targeting*. The above policy narrative was nevertheless subjected to formal econometric tests, which took into account all other enumerated potential changes in regime. The breakpoint and forecast Chow tests we performed

<sup>18</sup>The motivation behind this particular policy target modification is explained by the Bank of England (see website) in the following way: "The new target of 2.5% was quite a significant change from the previous target of 2.5% or less. The Treasury felt there were uncertainties about the old target – was it 2.5% or less than 2.5%, and if so how much less?"

confirmed what we already expected. As can be seen in Panel A (with RPI and final real GDP gap data) and Panel B (with RPIX and final real GDP gap data) of Table 3, the structural breaks delimiting our sample are the two out of the five tested that are most supported by our data. More precisely, in 6 out of all 8 test statistic probability values with RPI data and, again, in 6 out of all 8 with RPIX data, we reject the null of no structural break at the 5% significance level.<sup>19</sup> We, therefore, continued to estimate over our two subsamples and to compare the results across them.

**4.1.3. Backward-Looking Taylor Rules.** A way to address the problems of endogeneity and, potentially, serial correlation usually encountered in original Taylor rule equations while still applying the OLS method is to estimate them with all regressors *lagged* and by also adding an additional *lagged* dependent variable. As we have discussed in section 2, such backward-looking Taylor rules may have different lag length. Yet most papers have found that lags of 1 or 2 are usually sufficient to capture the dynamics of such equations. Another common finding has been that backward-looking Taylor rules are problematic in terms of econometric output, in addition to the problems with theoretical underpinnings. This is what we would like to also confirm here, with our results based on UK most recent data.

Following Nelson (2000, 2001, 2003), we estimated a backward-looking Taylor rule where all variables enter with lag 1. The results – see Table 4 – are poor and do not lend themselves to a sensible economic interpretation, apart perhaps one: such backward-looking specifications could be discarded from further attention on both economic and econometric grounds.

Again trying the exercise Nelson (2000, 2001, 2003) performed, we reestimated with two lags for all variables and then progressively eliminated the insignificant terms. We could not arrive, however, at any reasonable parsimonious backward-looking Taylor rule, which is supportive of what we have just concluded.

**4.2. Two-Stage Least Squares: Classic and Backward-Looking Taylor Rules.** A second way to address the problems of endogeneity and, potentially, serial correlation in classic Taylor rule equations is to replace OLS by TSLS. That is what we did next.

**4.2.1. The Original (or Classic) Taylor Rule Again.** Our TSLS results enhanced, as a matter of fact, those from the OLS estimation reported above. Table 5 indicates that there is no any important difference in our conclusions, even in a quantitative aspect. First, such equations for the UK in the *post*-independence period of inflation targeting perform amazingly well and can be sensibly interpreted, similarly to the case of the original Taylor (1993) article. Second, they suggest that the output gap has not mattered *before* operational independence but has mattered afterwards, even *more* than inflation, if we judge by the magnitude of the respective coefficient estimates.

**4.2.2. Backward-Looking Taylor Rules Again.** Our TSLS estimation also confirmed what the OLS method had found earlier concerning *backward*-looking Taylor rules. That is why we end up by concluding that the poor performance of such equations is most likely due not to the econometric approach implemented but rather to their problematic justifiability from the perspective of economic theory. We, therefore, leave them aside as economically irrelevant (and econometrically weak) and turn next in greater detail to the theory behind *forward*-looking Taylor rules and to our results from estimating alternative specifications of such policy reaction functions.

**4.3. Generalized Method of Moments: Forward-Looking Taylor Rules.** In this paper, we essentially focus on the strategy of estimating forward-looking Taylor rules popularized by Clarida, Gali and Gertler (1997, 1998 a, b, 2000). Since the Clarida-Gali-Gertler approach also provides a solid *theoretical* rationale for our empirical work here, it is summarized next, using the less ambiguous notation we introduced.

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<sup>19</sup>It is true that there is also strong evidence for another structural change, in 1995:2: in 6 out of all 8 test statistic probability values with RPI data and in 5 out of all 8 with RPIX data we reject the null of no structural break. Yet we cannot split our pre-independence subsample into two additional subsamples, as the number of observations will be only about 10 or less in each. The fifth potential break, in 2004:1, is definitely discarded by our Chow tests. The test results for the first potential break, in 1990:4 (outside the sample of interest to us here), are rather mixed; overall, however, rejection findings on the null of no structural change dominate, so there is good reason to accept a break in 1990:4 as well, based on both econometric and descriptive evidence.



4.3.1. *Theoretical Rationale for Our Forward-Looking Specifications.* It is by now standard to think of monetary policy reaction functions in general, and of Taylor rules in particular, as if derived from an underlying model of the economy. This model is usually the baseline New Keynesian model described in King and Woolman (1996 a, b) and Yun (1996), among others, and also called – first by Goodfriend and King (1997) and in a broader context – the New Neoclassical Synthesis (NNS) model. It is not necessary for our purposes here to fully write down this model, as such sticky-price analytical frameworks have been well explored.<sup>20</sup> We would rather sketch its relevance to our estimation below, by simply stating its ‘core’ equations and then relating them to the forward-looking feedback rules we estimated by applying GMM.

After log-linearization around a zero inflation steady state, the equilibrium conditions of the baseline New Keynesian (or NNS) model are embodied in four equations, which – following Clarida, Galí and Gertler (2000) in ignoring certain constant terms, but using our notation here – can be written as:

$$(4.1) \quad \pi_t = \delta E[\pi_{t+1} | \mathcal{I}_t] + \lambda(y_t - \xi_t),$$

$$(4.2) \quad y_t = E[y_{t+1} | \mathcal{I}_t] - \frac{1}{\sigma}(i_t - E[\pi_{t+1} | \mathcal{I}_t]) + \zeta_t,$$

$$(4.3) \quad i_t^T = \beta_{\pi,+1} E[\pi_{t+1} | \mathcal{I}_t] + \beta_{x,0} x_t,$$

$$(4.4) \quad i_t = \beta_{i,-1} i_{t-1} + (1 - \beta_{i,-1}) i_t^T.$$

Equation (4.1) is a (New Keynesian) forward-looking Phillips curve, or also alternatively termed a (New Keynesian) forward-looking aggregate supply (AS) curve.  $\mathcal{I}_t$  is the information set available at time  $t$ .  $\delta$  is the discount factor and  $\lambda$  the output elasticity of inflation.  $y_t \equiv \ln Y_t$  is the current-period level of output and  $\xi_t$  is the natural rate of output, defined as the level of output that would obtain under fully flexible prices and assumed to follow an AR(1) process. This AS curve can be derived by aggregation of optimal price-setting decisions by monopolistically competitive firms under Calvo (1983) individual price adjustment.

(4.2) is interpreted as a (New Keynesian) forward-looking IS curve, and is derived as a combination of a standard consumption Euler equation and a market clearing condition.  $\sigma$  denotes the coefficient of relative risk aversion (CRRA) embedded in the utility function.  $\zeta_t$  is in this context usually interpreted as an exogenous demand shock: like  $\xi_t$  in the aggregate supply curve (4.1), it is assumed to follow an AR(1) process.

Equation (4.3) is a (New Keynesian) forward-looking monetary policy rule of the usual Taylor type.

(4.4), finally, is a (New Keynesian) interest rate smoothing equation.

In our notation, all *policy* parameters are easily recognized by the  $\beta$ -letter: each subscript to it consists of a pair of symbols, the first being the respective letter designating the variable to which the  $\beta$ -coefficient relates and the second being a positive or negative integer denoting the respective lead (+) or lag (–), with 0 standing for the current period. We use this notation further down in the text and tables for a clearer reference.

Following the above New Keynesian approach, one can summarize the policy of the central bank by a linear instrument rule of the Taylor type and with forward-looking formation of rational expectations:

$$(4.5) \quad i_t^T = i^T + \beta_{\pi,+k} (E[\pi_{t+k} | \mathcal{I}_t] - \pi^T) + \beta_{x,+q} E[x_{t+q} | \mathcal{I}_t],$$

where, as noted,  $\mathcal{I}_t$  is the information set available to the monetary authority at the time the interest rate is set. By construction,  $i^T$  is the desired (constant) nominal interest rate when inflation is at its target level and output is at potential. (4.5) is the empirical counterpart of (4.3) above. Clarida-Galí-Gertler (1997, 1998 a, b, 2000) claim in a series of articles that such a monetary policy reaction function has some appeal on both theoretical and empirical grounds. Theoretically, approximate forms of rules like (4.3) are optimal if the monetary authority has as objective a quadratic loss function in deviations of inflation and output from their respective targets in the context of the New Keynesian macromodel we have just outlined. Empirically, rules like (4.5) usually provide a reasonably good summary of most central banks’ behavior in recent years.

<sup>20</sup>An excellent textbook source is Woodford (2003)

Adding and subtracting  $E[\pi_{t+k} | \mathcal{I}_t] - \pi^T$  to the RHS of (4.5),

$$i_t^T = i^T + \beta_{\pi,+k} (E[\pi_{t+k} | \mathcal{I}_t] - \pi^T) +$$

$$+ \beta_{x,+q} E[x_{t+q} | \mathcal{I}_t] + (E[\pi_{t+k} | \mathcal{I}_t] - \pi^T) - (E[\pi_{t+k} | \mathcal{I}_t] - \pi^T),$$

and rearranging,

$$\underbrace{i_t^T - E[\pi_{t+k} | \mathcal{I}_t]}_{\equiv r_{t,+k}^T} = i^T + \beta_{\pi,+k} (E[\pi_{t+k} | \mathcal{I}_t] - \pi^T) + \\ + \beta_{x,+q} E[x_{t+q} | \mathcal{I}_t] - \pi^T - (E[\pi_{t+k} | \mathcal{I}_t] - \pi^T),$$

implies an ex ante *real* interest rate *target* that can be written as:

$$(4.6) \quad r_{t,+k}^T = \underbrace{(i_t^T - \pi^T)}_{\equiv r^* = \text{const}} + (\beta_{\pi,+k} - 1) (E[\pi_{t+k} | \mathcal{I}_t] - \pi^T) + \beta_{x,+q} E[x_{t+q} | \mathcal{I}_t].$$

Clarida-Gali-Gertler (1998 b, 2000) point to the insights embodied in (4.6); it clearly shows that (i) attaining the target ‘on average’ and assuming that the real interest rate is determined by non-monetary factors in the long run implies a constraint on  $i^T$  which should be set equal to the exogenously given long-term ‘equilibrium’ real interest rate  $r^*$  plus the inflation target  $\pi^T$ ; (ii) interest rate rules characterized by  $\beta_\pi > 1$  and  $\beta_x > 0$  will tend to be stabilizing, to the extent that lower real interest rates boost economic activity but also inflation, while those with  $\beta_\pi < 1$  and  $\beta_x < 0$  will tend to be destabilizing.

Incorporating interest rate smoothing behavior, widely supported by the practice of central banks as well as from a theoretical perspective, and allowing for exogenous interest rate (i.e., here also monetary policy) shocks requires an extension of the interest rate *target* (4.5) by also specifying a model for the *actual* interest rate:

$$(4.7) \quad i_t = \beta_i(L) i_{t-1} + (1 - \beta_{i,-1}) i_t^T + \nu_t.$$

In (4.7),  $L$  denotes the lag operator,  $\beta_i(L) \equiv \beta_{i,-1} + \beta_{i,-2}L^1 + \dots + \beta_{i,-n}L^{n-1}$  with  $\beta_{i,-1} \in [0, 1]$  measures the degree of smoothing of interest rate changes and  $\nu_t$  is a zero mean exogenous interest rate shock. (4.7) is, in turn, the empirical counterpart of (4.4) above.

Plugging the Taylor rule target (4.5) into the partial adjustment model (4.7),

$$(4.8) \quad i_t = \beta_i(L) i_{t-1} + \\ + (1 - \beta_{i,-1}) \{i_t^T + \beta_{\pi,+k} (E[\pi_{t+k} | \mathcal{I}_t] - \pi^T) + \beta_{x,+q} E[x_{t+q} | \mathcal{I}_t]\} + \nu_t,$$

representing the expected values,  $E[\pi_{t+k} | \mathcal{I}_t]$  and  $E[x_{t+q} | \mathcal{I}_t]$ , respectively, as realized values minus forecast errors,  $\pi_{t+k} - (\pi_{t+k} - E[\pi_{t+k} | \mathcal{I}_t])$  and  $x_{t+q} - (x_{t+q} - E[x_{t+q} | \mathcal{I}_t])$ , respectively,

$$i_t = \beta_i(L) i_{t-1} + (1 - \beta_{i,-1}) \times$$

$$\times \{i_t^T + \beta_{\pi,+k} ([\pi_{t+k} - (\pi_{t+k} - E[\pi_{t+k} | \mathcal{I}_t])] - \pi^T) + \beta_{x,+q} [x_{t+q} - (x_{t+q} - E[x_{t+q} | \mathcal{I}_t])]\} + \nu_t,$$

and rearranging, yields an equation for the *actual* (not target) nominal interest rate of the form

$$(4.9) \quad i_t = (1 - \beta_{i,-1}) \left\{ \underbrace{[r^* - (\beta_{\pi,+k} - 1) \pi^T]}_{\equiv \beta_{0,+k}} + \beta_{\pi,+k} \pi_{t+k} + \beta_{x,+q} x_{t+q} \right\} + \underbrace{\beta_i(L) i_{t-1}}_{\equiv \beta_{i,-1}} + \varepsilon_t,$$

where

$$(4.10) \quad \varepsilon_t \equiv (1 - \beta_{i,-1}) \{ \beta_{\pi,+k} (\pi_{t+k} - E[\pi_{t+k} | \mathcal{I}_t]) + \beta_{x,+q} (x_{t+q} - E[x_{t+q} | \mathcal{I}_t]) \} + \nu_t.$$

It can be seen in (4.10) that  $\varepsilon_t$  is a linear combination of *forecast errors* of inflation and the output gap (the term in curly brackets) and the exogenous disturbance to the interest rate,  $\nu_t$ : it is thus orthogonal

to any variable in the information set  $\mathcal{I}_t$  available at time  $t$ . Now let  $\mathbf{z}_t$  denote a vector of variables within the central bank's information set at the time when the decision on the interest rate is made, that is,  $\mathbf{z}_t \in \mathcal{I}_t$ . As Clarida-Gali-Gertler (1998 b), p. 1039, suggest, possible elements of  $\mathbf{z}_t$  (and, thus, instruments in the econometric sense) include any lagged variables that help forecast inflation and output, as well as any contemporaneous variables that are uncorrelated with the current-period interest rate shock  $\nu_t$ . Since  $E[\varepsilon_t | \mathbf{z}_t] = 0$ , (4.9) then implies the set of *orthogonality* conditions

$$(4.11) \quad E \left[ \left\{ i_t - (1 - \beta_{i,-1}) \left( \underbrace{[r^* - (\beta_{\pi,+k} - 1) \pi^T]}_{\equiv \beta_{0,+k}} + \beta_{\pi,+k} \pi_{t+k} + \beta_{x,+q} x_{t+q} \right) - \beta_i(L) i_{t-1} \right\} \mathbf{z}_t \right] = 0.$$

These orthogonality conditions provide the basis for the estimation of the parameters of interest, collected in the vector  $\beta' \equiv (\beta_{0,+k}, \beta_{\pi,+k}, \beta_{x,+q}, \beta_{i,-1})'$ , applying the Generalized Method of Moments (GMM) first suggested by Hansen (1982). Standard errors were calculated on the basis of the delta method, as described in detail in Appendix B. Clarida, Galí and Gertler (1998 b, 2000) note that, by construction, the first component of  $\{\varepsilon_t\}$  follows an  $MA(a)$  process, with  $a = \max\{k, q\} - 1$  and will thus be serially correlated unless  $k = q = 1$ . For that reason, the GMM estimation should be carried out with a weighting matrix that is robust to autocorrelation (and heteroskedasticity), as we also do. Moreover, to the extent that the dimension of vector  $\mathbf{z}_t$  is *higher* than the number of parameters to estimate, 4 in our case here, (4.11) implies some *overidentifying* restrictions that can be tested in order to assess the validity of the specification estimated as well as the set of instruments used. We present such test statistics in the tables of results reported in Appendix A and discussed further down. The test rests on the following logic, as exposed in Clarida, Galí and Gertler (1998 b), pp. 1040-1041. Under the *null*, the central bank adjusts the interest rate each period so that (4.8) holds, with the RHS expectations based on all relevant information available to policymakers at that time (which includes  $i_t$  itself, since expected inflation and output will not be invariant to it). With the Clarida-Gali-Gertler assumptions, of which we also make use here, that implies the existence of values for the vector  $\beta$  such that the residual  $\varepsilon_t$  is orthogonal to the variables in the information set  $\mathcal{I}_t$ . Under the *alternative*, the central bank adjusts the interest rate in response to changes in some current and/or lagged variables, but not necessarily in connection with the information that those changes contain about future inflation and output. In that case, some relevant explanatory variables are being omitted from (4.9). To the extent that some of those variables are correlated with the instruments  $\mathbf{z}_t$ , the set of orthogonality conditions (4.11) will be violated, which would lead to a statistical rejection of the model (given a sufficiently large sample).

**4.3.2. Summary of Estimates of Forward-Looking Taylor Rules.** Let us now look in more detail at our key results from estimating *forward-looking* Taylor rules via GMM. We have estimated specifications where the *lead* for inflation varies from 1 to 8 quarters ahead,  $k = 1, \dots, 8$ , and that for the output gap from 0 to 4,  $q = 0, \dots, 4$ . For the output gap, both *final* (or *revised*) and *real-time* (or *initial*) GDP data have been used and specifications with both *quadratic* and *Hodrick-Prescott* filtering to obtain the output gap have been estimated. For the price level, underlying the definition of inflation, both the *RPI* and the *RPIX* have been alternatively employed. Moreover, the RPI, the RPIX and the 3-month Treasury bill enter our various specifications either as they were released from the original data sources and thus *not seasonally adjusted* (nsa) or *seasonally adjusted* (sa) by the Census X12 procedure. All these numerous estimates are summarized in the tables collected in Appendix A, which we briefly discuss next.<sup>21</sup>

Panels A and B in Table 6 compare the coefficients of interest here from an identical forward-looking Taylor rule estimated over the *pre-* and *post-independence* subsamples, respectively, using the RPI to calculate inflation and both final and real-time GDP gap data. Panels A and B in Table 7 do the same for the RPIX instead; we now report results only for the *real-time* GDP gap series, since when *revised* GDP data were used the regressions were not easily interpretable. Tables 8 and 9 report our corresponding estimates when the price level used to define inflation and the 3-month Treasury bill rate have first been *deseasonalized* before entering the regressions.

The leads of  $k = 2, 3$  for inflation and of  $q = 0, 1$  for the output gap were selected as the preferred ones across all attempted specifications from the viewpoint of both econometrics and economics. In the former case, the econometric characteristics of the regressions such as statistical significance of most relevant parameters, higher adjusted R-squared, lower standard error of regression (SER) and higher probability

<sup>21</sup>All other details, including the data files and the EViews programs, are available upon request.

value of the Hansen J-test for the validity of overidentifying restrictions have mattered overall. In the latter case, the signs and magnitudes of the statistically significant monetary policy feedback coefficients to inflation and to the output gap and the value of the interest rate smoothing parameter have been selected across our multiple specifications so as to make most economic sense and to allow reasonable interpretation. As can be verified in the last row of tables 6 and 7, the validity of our overidentifying restrictions and of the set of our instruments cannot be rejected for all equations we report and the goodness of fit is also very high (especially for the second, later subsample, perhaps partly because of the greater number of observations). Then, the coefficients of interest are statistically significant at all conventional levels in all specifications. Moreover, the positive expected signs of the response to both inflation and the output gap and the bounds between 0 and 1 of the smoothing parameter are always and everywhere (in the 16 specifications on which we focus attention here) satisfied. The respective tables with *seasonally adjusted* price level and interest rate, 8 and 9, are not that perfect econometrically.

Finally, we look into the magnitudes of the reaction coefficients. We would like to stress, in particular, the numerical ranges obtained with RPIX (*nsa*) and *real-time* GDP data (i.e., Table 7), mostly because such were the data of relevance and at the disposal to the monetary authority when making policy decisions reflected in changes in the interest rate. However, discussing any of the other reported specifications (in Table 6 as well as in most other tables in Appendix A) will not substantially modify the *quantitative essence* of our major findings. These are as follows.

Reaction to Inflation. To begin with, and what appeared to us at first sight surprising, is that most reported estimates confirm that the *positive* magnitude of the coefficient to inflation has *declined* (or even become in some specifications statistically *insignificant*) in the post-independence subsample relative to the pre-independence one.

In Table 6, for example, where the *RPI* is used to calculate inflation,  $\beta_{\pi,+2}$  has dropped from 1.09 in the *pre*-independence subsample to 0.48 in the *post*-independence one if quadratic detrending of *real-time* real GDP is used to obtain the gap measure, or from 0.91 to 0.73 if Hodrick-Prescott filtering is applied instead. If we take the corresponding results with *seasonally adjusted* RPI and interest rate in Table 8, the quadratic specification shows again a drop from 0.80 to 0.61 yet the Hodrick-Prescott one results in a rise from 0.71 to 0.91.

Looking now at the same results but when the *RPIX* is used instead to compute the rate of inflation, as reported in the last two columns of Table 7, the quadratic specification indicates a (relatively) moderate fall of  $\beta_{\pi,+2}$  from 0.67 to 0.50 and the Hodrick-Prescott one from 0.81 to 0.50 (and, moreover, the corresponding reduced-form coefficient,  $b_{\pi,+2}$ , is marginally insignificant at the 10% level). If we take the respective results with *seasonally adjusted* RPIX and 3-month Treasury bill rate in Table 9, these conclusions are now reversed: the quadratic specification shows a (relatively) sharp rise from 0.10 to 0.59 and the Hodrick-Prescott one from 0.89 to 2.32. If one considers, in turn, the first pair of columns in Table 7, where the inflation lead (again, RPIX-based) is specified at 3 instead of 2 quarters, the quadratic version registers similarly a fall of  $\beta_{\pi,+3}$  from 1.45 to 1.07 but the Hodrick-Prescott version now shows a considerable rise from 1.01 to 1.83. No matter the disparity, it is worth noting here that all four policy responses to inflation estimated on RPIX-lead of 3 quarters and current-period real-time GDP gap are consistent in quantitative terms with the theoretically expected values of above 1. In this sense, these could represent our ‘most reasonable’ quantification of Bank of England’s feedback to inflation. The magnitudes are close to what Nelson (2001), p. 20, tables 5 and 6, reported, 1.27, as his preferred estimate for the pre-independence inflation targeting subsample.<sup>22</sup> On the other hand, Clarida, Galí and Gertler (1998 b), p. 1055, Table 4, come up with an inflation response estimate for their 1979:06-1990:10 monthly British sample marginally lower than unity, namely 0.98, which is more in line with most of the estimates we reported above.<sup>23</sup>

Thus, our findings on the reaction of the UK monetary authorities to inflation before and after operational independence are somewhat sensitive to the data and specifications used and, therefore, inconclusive. If one can justify that (i) *real-time* GDP data matter and not final ones, (ii) *RPIX* is more relevant throughout our sample period than RPI, and (iii) for consistency with the other variables entering the Taylor rule regressions, prices and interest rates should be *seasonally adjusted*, a stronger

<sup>22</sup>For this last subsample of quarterly data in his study, coinciding exactly with our pre-independence inflation targeting subsample, Nelson (2001), p. 20, Table 5, estimates via TSLS Bank of England’s response to the output gap at 0.47 and interest rate smoothing coefficient at 0.29.

<sup>23</sup>For their sample, earlier than ours and containing shifts in monetary regime, Clarida, Galí and Gertler (1998 b), p. 1055, Table 4, quantify via GMM Bank of England’s response to the output gap at 0.19 and interest rate smoothing coefficient at 0.92.

response to inflation in the post-independence period by the Bank of England is what our estimates largely confirm. Yet if one of these conditions is not well-substantiated, it may well be that the reaction to inflation has been weakened relative to pre-independence inflation targeting.

**Interest Rate Smoothing.** In a similar way, we cannot say much as to whether the degree of interest rate smoothing, reflected in our estimates for the coefficient,  $b_{i,-1}$ , has become stronger or weaker after the Bank of England was granted operational independence from HM Treasury in May 1997. Evidently, any conclusion in this sense would rest on a restrictive interpretation of a subset of our Taylor rule specifications and alternative proxies, which we would not wish to force on the data. It might also well be that, with respect to both inflation and interest rate smoothing, the post-independence behavior of the UK monetary authority has not changed much, if at all, and for that reason cannot be definitely detected and confirmed by our data.

**Reaction to the Output Gap.** Yet if our results on the reaction of the Bank of England to inflation and on its approach to interest rate smoothing cannot lead to any well-grounded conclusion, the more so in a quantitative context, we believe that the contribution of the present empirical work is mostly in uncovering an initially unexpected but after all logically interpretable central bank response to the output gap, fully consistent with an inflation targeting strategy given the stage in the business cycle. We elaborate on that below.

All 32 cells in our tables 6 through 9 except one (in the *post*-independence Hodrick-Prescott column for final real GDP gap data in Table 8, the only estimate as well that is not statistically significant at the 1% and the 5% levels) report always (i) *statistically significant* and (ii) *positive* estimates for the coefficient to the contemporaneous output gap,  $b_{x,0}$ , which, most importantly, (iii) indicate a *unanimous and considerable rise* in the magnitude of the corresponding structural-form parameter,  $\beta_{x,0}$ , in the *post*-independence period! This is therefore a robust finding that at first puzzled us a bit: why should a central bank in an inflation targeting regime increase its reaction to the output gap after receiving operational independence, with its reaction to inflation at the same time most likely not much changed or, if increased, not at a comparable degree? Well, to see why this is exactly what a central bank whose priority is to keep inflation low should do, the more so under inflation targeting, one needs to take into consideration the stage of the business cycle as well, in particular before and after operational independence.

**4.3.3. Asymmetric Monetary Policy Response Across the Business Cycle.** The easiest way to understand such logic is to look at the (dominant) phase of the business cycle before and after operational independence. Comparing the descriptive statistics in panels A and B of Table 1 in Appendix A, one can find that the mean output gap in 1992:4-1997:1 has been of the order of  $-0.38\%$  (Hodrick-Prescott measure) to  $-1.24\%$  (quadratic measure) if *final* GDP data are used and of the order of  $-0.34\%$  (Hodrick-Prescott measure) to  $-0.97\%$  (quadratic measure) with *real-time* GDP data instead; the same statistic for the period of operational independence, 1997:3-2004:4 (or 2001:4 for *real-time* GDP data) is, respectively, of the order of  $0.07\%$  (Hodrick-Prescott measure) to  $0.16\%$  (quadratic measure) if *final* GDP data are used and of the order of  $0.11\%$  (Hodrick-Prescott measure) to  $-0.05\%$  (quadratic measure) with *real-time* GDP data. Similar conclusions can be inferred from figures 7 and 8 in Appendix A. This dimension of our analysis makes a clear point: the Bank of England has reacted in a much stronger way to the output gap when aggregate demand has, on average, been close(r) to potential, thus creating inflationary pressures, i.e., (mostly) during the post-independence period.

**4.3.4. Additional Robustness Checks.** To check this empirically identified asymmetry in the magnitude (not the sign) of the central bank's response to the output gap depending on the business cycle, we have proceeded to estimations of the same Taylor rule specifications but with real GDP *growth* replacing real GDP *gap*. This is also another way of continuing our extensive sensitivity analysis here, embodied in the various specifications and proxies involved.

**Has the Bank Reacted to Real GDP Growth?** So, during operational independence the Bank of England has reacted more aggressively to the output *gap* relative to the pre-independence inflation targeting period. This is true (i) no matter how the output gap is measured; (ii) no matter the alternative proxies for all variables included in our forward-looking Taylor rules; (iii) and no matter the several lead specifications estimated that make sense from both a statistical and an economic point of view.

But has the Bank also reacted in a similar way to real GDP *growth*? Table 10 presents evidence that what the Bank of England has really cared about throughout the inflation targeting period is the output gap and not the rate of growth of real output: nowhere in this table, before as well as after operational independence, is the coefficient on real GDP growth statistically significant at all. The UK data thus

clearly reject the idea that real GDP *growth* should enter Taylor rules in place of the real GDP *gap*, as some authors have suggested (often because of the measurability problems inherent in the output gap definition).

And there is good economic logic behind such a finding. It can be summarized in the following way. There is no need for a central bank to (aggressively) react to any change in the rate of *growth* of real GDP *per se*; for example, real expenditure may grow in a depressed economy and there is no need to overhastily fight such a (stabilizing) tendency. It is only with respect to a benchmark *potential* output that the increase in aggregate demand should matter for inflationary expectations, and hence for an inflation-targeting central bank. But once aggregate expenditure comes close to the estimated capacity of an economy to produce output and threatens to surpass it, thus creating inflationary pressure and affecting unfavorably the rational expectations of economic agents about future inflation, the central bank should respond (aggressively), the more so under an inflation targeting framework. That is our major, and we would hope, novel interpretation of the empirical findings in the present paper. It furthermore confirms that the Bank of England has reacted flexibly, in a justified and consistent way, to the changing business cycle conditions in the period of its operational independence relative to the pre-independence inflation targeting period, as also implied by its mandate.

What about an Explicit Exchange Rate Term? As a last robustness check of our findings, we also performed Taylor rule regressions with an explicit exchange rate term, namely the nominal effective exchange rate (NEER) index, added to the standard variables in (2.5). As one can verify in Table 11, a general conclusion when including the NEER into our alternative specifications is that it is, as a rule, statistically significant but of a very negligible magnitude, practically close to zero, and with an uncertain – that is, switching across specifications – sign. More importantly, the inclusion of the NEER also makes all policy responses unrealistically low, the more so during the operational independence period, while at the same time pushing the interest rate smoothing parameter and, especially, the adjusted  $R^2$  for the regressions conspicuously high, which is indicative of a likely misspecification. For this reason we avoid here any further comments on our forward-looking Taylor rules with an explicit exchange rate term.

## 5. CONCLUDING COMMENTS

This paper contributed to the empirical investigation of whether increased central bank independence matters for the conduct of monetary policy, in particular within a broader and stable inflation targeting framework. We took advantage of the unique experience in that sense of the United Kingdom: the Bank of England was granted operational independence from HM Treasury in May 1997, whereas inflation targeting had been effective since October 1992. Our econometric strategy focused on estimating forward-looking Taylor rules using the GMM approach, theoretically consistent with the New Keynesian monetary policy model recently popularized in similar contexts by Clarida, Gali and Gertler. Yet we also applied OLS and TSLS to classic and backward-looking Taylor rules, for the purpose of comparisons with earlier work as well as across our alternative econometric techniques, regression specifications and variable proxies.

An interesting finding from our empirical study is that *classic* Taylor rules perform impressively in describing (ex post) UK monetary policy throughout the period of *operational independence*; moreover, such rules indicate that the Bank of England has definitely responded to the output gap *after* independence but *not before*. However, because of the criticisms to equations this type, we place predominant weight and a substantial confidence of robustness on our extensive results from estimating via GMM forward-looking feedback rules incorporating interest rate smoothing. These results generally confirm what OLS and TSLS have partly uncovered earlier.

One principal lesson we extract from UK quarterly data is that, perhaps astonishingly at first sight, the Bank of England has reacted *much more aggressively* to the output gap *after* receiving independence. This *asymmetry* of the monetary policy reaction function remains robust across a number of Taylor rule specifications and alternative proxies for the explanatory variables. We argue that it is fully consistent with an inflation targeting central bank when the stage of the *business cycle* is also taken into consideration. Our main contribution is to present evidence that the Bank of England has responded to the output *gap*, the more so in periods when it is positive or close to zero and when inflation is at the same time credibly stabilized around target, and *not* at all to output *growth*. This is exactly how it should be according to New Keynesian theory, in particular under inflation targeting when the primary (if not the only) concern of the central bank is to keep low and stable inflation. The monetary authority should care (for theoretical reasons), and did seem to care (in our empirical results), not whether aggregate expenditure on output grows *per se*, but whether such growth implies – as would be in a stage of the

business cycle *above* or *close to* potential output – increasing inflationary pressure or not. As far as the magnitude of the reaction of the Bank of England to inflation and its degree of interest rate smoothing are concerned, we do not find overwhelming evidence to conclude in favor of any important, substantial change across our two subsamples, pre- and post- operational independence.

Finally, to address in a summary the question we posed as a title to this study, we were not able to arrive at a definite answer to it either. On the one hand, we here presented estimates that identify a different response of the Bank of England to the output gap, much stronger after it was granted operational independence from HM Treasury. However, we also paid attention to the facts that – as the new millennium was approaching and gradually unfolding in the UK – the stage in the business cycle had progressively changed, passing from a recessionary to an expansionary phase, and that inflation had been anchored at target. Given our short sample, containing roughly one full cycle of contraction and recovery of the British economy within the inflation targeting period on which we focused, we were not in a position to disentangle the individual contribution of each of the three principal factors we pointed to as underlying, and largely explaining, the change in the Bank of England's reaction function. A provisional answer to our title could, therefore, be that the expanding economy and the anchored inflation (and, hence, inflation expectations) have mattered at least as much, if not more, as greater central bank autonomy in implementing the British inflation targeting strategy. There is thus much room remaining for further research along these lines, within the UK context as well as across other nations and monetary regimes.

## 6. APPENDIX A: DATA AND ESTIMATION OUTPUT

PANEL A: Pre-Independence Subsample: 1992:4 – 1997:1									
	Inflation, % p.a.		RGDP growth, % p.a.		3-m TBill rate, % p.a.	RGDP gap, % of potential			
	RPI	RPIX	Fin data	RT data		HP Fin	Q Fin	HP RT	Q RT
Mean	2.48	2.80	2.92	2.98	5.79	−0.38	−1.24	−0.34	−0.97
Median	2.54	2.81	2.84	2.77	5.76	0.04	−0.73	0.02	−0.49
Max	3.60	3.62	4.68	4.97	6.73	0.65	−0.11	0.95	0.20
Min	1.29	2.17	0.99	0.87	4.95	−2.15	−3.09	−2.35	−2.83
SD	0.65	0.35	1.03	1.13	0.58	0.97	1.04	1.06	1.10
J-B p-v	0.79	0.81	0.94	0.82	0.57	0.23	0.26	0.35	0.29
# obs	18	18	18	18	18	18	18	18	18
PANEL B: Post-Independence Subsample: 1997:3 – 2004:4 (or 2001:4 for real-time GDP data)									
	Inflation, % p.a.		RGDP growth, % p.a.		3-m TBill rate, % p.a.	RGDP gap, % of potential			
	RPI	RPIX	Fin data	RT data		HP Fin	Q Fin	HP RT	Q RT
Mean	2.50	2.34	2.74	2.71	5.09	0.07	0.16	0.11	−0.05
Median	2.66	2.27	2.73	2.82	4.85	0.10	0.24	0.15	−0.05
Max	3.94	2.90	4.33	3.55	7.50	0.90	1.60	0.61	0.64
Min	1.04	1.85	1.52	1.63	3.50	−0.85	−1.25	−0.68	−1.35
SD	0.83	0.31	0.71	0.59	1.20	0.46	0.87	0.37	0.50
J-B p-v	0.36	0.40	0.69	0.57	0.35	0.70	0.43	0.50	0.35
# obs	30	30	30	30	30	30	30	18	18

TABLE 1. Descriptive Statistics of the Data

EXPLANATORY NOTE TO TABLE 1: All data are quarterly and for the United Kingdom; RPI = Retail Price Index; RPIX = RPI eXcluding the mortgage rate; RGDP = Real GDP; 3-m TBill rate = 3-month Treasury Bill rate; Fin = final (data); RT = real-time (data); HP = Hodrick-Prescott (detrending); Q = quadratic (detrending); SD = standard deviation; J-B p-v = Jarque-Bera statistic probability value (for testing the null of normality of regression residuals); # obs = number of observations.



PANEL A: Pre-Independence Subsample: 1992:4 – 1997:1 (18 observations)				
Real GDP Filter:	Quadratic	Hodrick-Prescott	Quadratic	Hodrick-Prescott
$i^T$	5.77*** (0.15)	5.76*** (0.09)	6.03*** (0.23)	5.78*** (0.13)
$b_{0,0}$	3.88*** (0.49)	3.68* (0.41)	5.00*** (0.68)	4.58*** (0.66)
$b_{\pi,0}$	0.75*** (0.16)	0.83*** (0.15)	0.41* (0.21)	0.48* (0.42)
$b_{x,0}$	-0.03 (0.10)	-0.12 (0.10)	0.27 (0.23)	0.20 (0.33)
$AR1$ term			0.44*** (0.15)	0.42*** (0.21)
Adj $R^2$	0.63	0.66	0.75	0.72
SER	0.35	0.34	0.29	0.31
DW	0.90	1.07	$AR1$ correction	$AR1$ correction
F p-v	0.000224	0.000118	0.000046	0.000101
PANEL B: Post-Independence Subsample: 1997:3 – 2004:4 (30 observations)				
Real GDP Filter:	Quadratic	Hodrick-Prescott	Quadratic	Hodrick-Prescott
$i^T$	4.95*** (0.14)	5.01*** (0.17)	4.56*** (0.75)	4.18*** (1.22)
$b_{0,0}$	2.81*** (0.45)	3.38*** (0.56)	3.55*** (0.81)	3.22** (1.27)
$b_{\pi,0}$	0.86*** (0.17)	0.65*** (0.21)	0.40*** (0.14)	0.38*** (0.14)
$b_{x,0}$	0.85*** (0.16)	0.38** (0.43)	0.37 (0.24)	0.37 (0.23)
$AR1$ term			0.90*** (0.08)	0.93*** (0.06)
Adj $R^2$	0.61	0.39	0.92	0.91
SER	0.75	0.94	0.35	0.35
DW	0.39	0.22	$AR1$ correction	$AR1$ correction
F p-v	0.000001	0.000520	0.000000	0.000000

TABLE 2. Classic Taylor Rules: OLS Estimates on RPI and Final Real GDP Gap

EXPLANATORY NOTE TO TABLE 2: All data are quarterly and for the United Kingdom; the method of estimation is OLS; the estimated equations are (2.4) and (2.5), with intercepts  $i^T$  and  $b_{0,0}$ , respectively, and all other parameters the same, as explained in the main text; standard errors for the directly estimated coefficients ( $i^T$  and the  $b$ 's) are in parentheses; \*\*\*, \*\*, \* = statistical significance at the 1, 5, 10% level, respectively;  $AR1$  = correction for an autoregressive process in the error of the regression of order 1; Adj  $R^2$  = adjusted  $R^2$ ; SER = standard error of regression; DW = Durbin-Watson statistic (for testing first-order serial correlation in the error process when there is no  $AR1$  correction for it or lagged dependent variable in the regression specification); F p-v = F-statistic probability value (for the joint significance of all estimated parameters).

Panel A: RPI	Chow Breakpoint Test		Chow Forecast Test	
Real GDP Filter:	Quadratic	Hodrick-Prescott	Quadratic	Hodrick-Prescott
First Quarter after ERM Entry: 1990:4				
F-statistic p-v	0.003453	0.251725	0.000004	0.216954
Log Likelihood R p-v	0.001965	0.000000	0.000001	0.000000
Last Quarter before Pre-Independence Subsample: 1992:3				
F-statistic p-v	0.000000	0.178858	0.000000	0.111828
Log Likelihood R p-v	0.000000	0.000000	0.000000	0.000000
Quarter of Change from Band to Point Inflation Target: 1995:2				
F-statistic p-v	0.000002	0.348599	0.000006	0.465473
Log Likelihood R p-v	0.000000	0.002369	0.000002	0.007102
Quarter of Granting Operational Independence: 1997:2				
F-statistic p-v	0.000000	0.118577	0.000013	0.193673
Log Likelihood R p-v	0.000000	0.001745	0.000004	0.005389
First Quarter after Change from RPIX to CPI Target: 2004:1				
F-statistic p-v	0.089051	0.166717	0.050507	0.101868
Log Likelihood R p-v	0.069090	0.130820	0.037082	0.075381
Panel B: RPIX	Chow Breakpoint Test		Chow Forecast Test	
Real GDP Filter:	Quadratic	Hodrick-Prescott	Quadratic	Hodrick-Prescott
First Quarter after ERM Entry: 1990:4				
F-statistic p-v	0.030295	0.903319	0.007957	0.904376
Log Likelihood R p-v	0.021174	0.000041	0.004898	0.000043
Last Quarter before Pre-Independence Subsample: 1992:3				
F-statistic p-v	0.021666	0.957068	0.000847	0.882228
Log Likelihood R p-v	0.014665	0.043251	0.000423	0.007572
Quarter of Change from Band to Point Inflation Target: 1995:2				
F-statistic p-v	0.006954	0.704496	0.005327	0.913390
Log Likelihood R p-v	0.004226	0.045529	0.003157	0.243711
Quarter of Granting Operational Independence: 1997:2				
F-statistic p-v	0.001912	0.314723	0.014014	0.674784
Log Likelihood R p-v	0.001030	0.017629	0.009100	0.152758
First Quarter after Change from RPIX to CPI Target: 2004:1				
F-statistic p-v	0.962004	0.986517	0.670773	0.818735
Log Likelihood R p-v	0.957198	0.983954	0.639928	0.791767

TABLE 3. Chow Tests on Classic Taylor Rules with Final Real GDP Gap Data

EXPLANATORY NOTE TO TABLE 3: All data are quarterly and for the United Kingdom; Chow breakpoint and forecast tests were performed on equation (2.5) estimated via OLS; F-statistic p-v = F-statistic probability value (for testing the null of no structural break); Log Likelihood R p-v = Log-Likelihood Ratio statistic probability value (for testing the null of no structural break).

PANEL A: Pre-Independence Subsample: 1992:4 – 1997:1 (18 observations)		
Real GDP Filter:	Quadratic	Hodrick-Prescott
$b_{0,-1}$	3.46*** (0.54)	2.72*** (0.55)
$b_{\pi,-1}$	-0.02 (0.20)	-0.14 (0.21)
$b_{x,-1}$	0.31*** (0.09)	0.35*** (0.09)
$b_{i,-1}$	0.48*** (0.15)	0.61*** (0.16)
Adj R <sup>2</sup>	0.75	0.77
SER	0.29	0.28
DW	lagged dep var	lagged dep var
F p-v	0.000048	0.000026
PANEL B: Post-Independence Subsample: 1997:3 – 2004:4 (30 observations)		
Real GDP Filter:	Quadratic	Hodrick-Prescott
$b_{0,-1}$	0.05 (0.35)	0.25 (0.37)
$b_{\pi,-1}$	-0.22* (0.12)	-0.11 (0.11)
$b_{x,-1}$	-0.26** (0.12)	-0.18 (0.19)
$b_{i,-1}$	1.10*** (0.09)	0.99*** (0.08)
Adj R <sup>2</sup>	0.90	0.88
SER	0.38	0.41
DW	lagged dep var	lagged dep var
F p-v	0.000000	0.000000

TABLE 4. Backward-Looking Taylor Rules: OLS Estimates on RPI and Final Real GDP Gap

EXPLANATORY NOTE TO TABLE 4: All data are quarterly and for the United Kingdom; the method of estimation is OLS; the estimated equation is (2.9), with  $N = 1$ ; standard errors for the directly estimated (short-run) coefficients (the  $b$ 's) are in parentheses; \*\*\*, \*\*, \* = statistical significance at the 1, 5, 10% level, respectively; standard errors for the indirectly estimated (long-run) coefficients (the  $\beta$ 's) are computed via the delta method (see Appendix B) but not reported here, due to likely misspecification, as discussed in the main text; Adj R<sup>2</sup> = adjusted R<sup>2</sup>; SER = standard error of regression; DW = Durbin-Watson statistic (for testing first-order serial correlation in the error process when there is no  $AR(1)$  correction for it or lagged dependent variable in the regression specification); lagged dep var = lagged dependent variable included in the regression; F p-v = F-statistic probability value (for the joint significance of all estimated parameters).

PANEL A: Pre-Independence Subsample: 1992:4 – 1997:1 (18 observations)				
Real GDP Filter:	Quadratic	Hodrick-Prescott	Quadratic	Hodrick-Prescott
$i^T$	5.74*** (0.16)	5.75*** (0.09)	6.20*** (0.38)	5.78*** (0.13)
$b_{0,0}$	3.76*** (0.53)	3.51*** (0.44)	5.62*** (1.13)	4.42** (1.58)
$b_{\pi,0}$	0.79*** (0.17)	0.89*** (0.16)	0.23 (0.32)	0.54 (0.61)
$b_{x,0}$	−0.05 (0.11)	−0.16 (0.11)	0.48 (0.45)	0.13 (0.75)
AR1 term			0.52** (0.19)	0.39 (0.48)
Adj R <sup>2</sup>	0.63	0.66	0.73	0.72
SER	0.35	0.34	0.30	0.31
DW	0.94	1.16	AR1 correction	AR1 correction
F p-v	0.000228	0.000106	0.000067	0.000108
PANEL B: Post-Independence Subsample: 1997:1 – 2004:4 (30 observations)				
Real GDP Filter:	Quadratic	Hodrick-Prescott	Quadratic	Hodrick-Prescott
$i^T$	4.94*** (0.14)	5.00*** (0.18)	4.66*** (0.62)	4.23*** (1.15)
$b_{0,0}$	2.77*** (0.45)	3.43*** (0.57)	3.63*** (0.70)	3.40*** (1.22)
$b_{\pi,0}$	0.87*** (0.17)	0.63*** (0.22)	0.41*** (0.15)	0.33** (0.15)
$b_{x,0}$	0.89*** (0.17)	1.24*** (0.42)	0.56 (0.33)	0.65* (0.32)
AR1 term			0.88*** (0.10)	0.92*** (0.07)
Adj R <sup>2</sup>	0.61	0.38	0.91	0.91
SER	0.75	0.96	0.36	0.36
DW	0.40	0.24	AR1 correction	AR1 correction
F p-v	0.000001	0.000402	0.000000	0.000000

TABLE 5. Classic Taylor Rules: TSLS Estimates on RPI and Final Real GDP Gap

EXPLANATORY NOTE TO TABLE 5: All data are quarterly and for the United Kingdom; the method of estimation is TSLS; the estimated equations are (2.4) and (2.5), with intercepts  $i^T$  and  $b_{0,0}$ , respectively, and all other parameters the same, as explained in the main text; standard errors for the estimated coefficients are in parentheses; \*\*\*, \*\*, \* = statistical significance at the 1, 5, 10% level, respectively; AR1 = correction for an autoregressive process in the error of the regression of order 1; Adj R<sup>2</sup> = adjusted R<sup>2</sup>; SER = standard error of regression; DW = Durbin-Watson statistic (for testing first-order serial correlation in the error process when there is no AR1 correction for it or lagged dependent variable in the regression specification); F p-v = F-statistic probability value (for the joint significance of all estimated parameters).

PANEL A: Pre-Independence Subsample: 1992:4 – 1997:1 (18 observations)				
Real GDP Data:	Final /Revised/		Real-Time /Initial/	
Real GDP Filter:	Quadratic	Hodrick-Prescott	Quadratic	Hodrick-Prescott
$b_{0,+2}$	1.53*** (0.13)	2.02*** (0.21)	1.13*** (0.16)	1.29*** (0.18)
$\beta_{0,+2}$	3.52 (0.31)	4.48 (0.47)	2.73 (0.40)	3.13 (0.43)
$b_{\pi,+2}$	0.38*** (0.03)	0.21*** (0.04)	0.45*** (0.04)	0.37*** (0.05)
$\beta_{\pi,+2}$	0.88 (0.08)	0.47 (0.09)	1.09 (0.09)	0.91 (0.11)
$b_{x,0}$	0.11*** (0.04)	0.21*** (0.03)	0.07*** (0.02)	0.12*** (0.02)
$\beta_{x,0}$	0.60 (0.08)	0.48 (0.07)	0.47 (0.05)	0.29 (0.06)
$b_{i,-1}$	0.56*** (0.01)	0.55*** (0.02)	0.59*** (0.01)	0.59*** (0.01)
Adj R <sup>2</sup>	0.75	0.76	0.83	0.71
SER	0.29	0.28	0.31	0.31
J-stat	0.302956	0.288247	0.282935	0.289121
OvId p-v	0.79	0.82	0.82	0.82

PANEL B: Post-Independence Subsample:				
	1997:3 – 2004:4 (28 observations)		1997:3 – 2001:4 (18 observations)	
Real GDP Data:	Final /Revised/		Real-Time /Initial/	
Real GDP Filter:	Quadratic	Hodrick-Prescott	Quadratic	Hodrick-Prescott
$b_{0,+2}$	0.11 (0.11)	−0.07 (0.15)	2.82*** (0.22)	2.14*** (0.20)
$\beta_{0,+2}$	0.43 (0.44)	−0.38 (0.76)	4.62 (0.37)	3.72 (0.34)
$b_{\pi,+2}$	0.46*** (0.02)	0.38*** (0.03)	0.30*** (0.03)	0.42*** (0.04)
$\beta_{\pi,+2}$	1.79 (0.08)	1.96 (0.14)	0.48 (0.05)	0.73 (0.06)
$b_{x,0}$	0.21*** (0.04)	0.17*** (0.05)	0.92*** (0.10)	1.06*** (0.13)
$\beta_{x,0}$	0.71 (0.16)	0.88 (0.25)	1.38 (0.16)	1.85 (0.23)
$b_{i,-1}$	0.74*** (0.02)	0.81*** (0.02)	0.39*** (0.03)	0.42*** (0.03)
Adj R <sup>2</sup>	0.92	0.92	0.94	0.92
SER	0.35	0.35	0.24	0.29
J-stat	0.217969	0.224985	0.159138	0.264661
OvId p-v	0.73	0.71	0.97	0.85

TABLE 6. Forward-Looking Taylor Rules: GMM Estimates on RPI

EXPLANATORY NOTE TO TABLE 6: All data are quarterly and for the United Kingdom; inflation is computed using the RPI; the method of estimation is GMM; the instrument set includes 4 lags of all (3) variables in the estimated equation, (4.9), with  $k = 2$  and  $q = 0$ ; standard errors for the directly estimated (reduced-form) coefficients (the  $b$ 's) in parentheses are calculated using a Newey-West weighting matrix robust to error autocorrelation and heteroskedasticity of unknown form; \*\*\*, \*\*, \* = statistical significance at the 1, 5, 10% level, respectively; standard errors for the indirectly estimated (structural-form) coefficients (the  $\beta$ 's) are computed via the delta method (see Appendix B); Adj R<sup>2</sup> = adjusted R<sup>2</sup>; SER = standard error of regression; J stat = J-statistic: equals the minimized value of the objective function in GMM estimation and is used, following Hansen (1982), to test the validity of overidentifying restrictions when there are more instruments than parameters to estimate, like in our case here (we have  $3 \times 4 + 1 = 13$  instruments, including the constant, to estimate 4 parameters, and so there are  $13 - 4 = 9$  overidentifying restrictions: under the null that the overidentifying restrictions are satisfied, the J-statistic times the number of regression observations is distributed asymptotically  $\chi^2(m)$  with degrees of freedom  $m$  equal to the number of overidentifying restrictions, 9 in our case); OvId p-v = probability value of the above-summarized Hansen test for  $m = 9$  overidentifying restrictions.

PANEL A: Pre-Independence Subsample: 1992:4 - 1997:1 (18 observations)				
Real GDP Filter:	Quadratic	Hodrick-Prescott	Quadratic	Hodrick-Prescott
$b_{0,+2}$ or $b_{0,+3}$	1.43*** (0.13)	2.13*** (0.24)	2.01*** (0.17)	1.38*** (0.19)
$\beta_{0,+2}$ or $\beta_{0,+3}$	2.04 (0.19)	3.10 (0.36)	4.58 (0.38)	3.49 (0.47)
$b_{\pi,+3}$	1.01*** (0.08)	0.69*** (0.17)		
$\beta_{\pi,+3}$	1.45 (0.12)	1.01 (0.24)		
$b_{\pi,+2}$			0.29*** (0.08)	0.32*** (0.05)
$\beta_{\pi,+2}$			0.67 (0.19)	0.81 (0.14)
$b_{x,0}$	0.20*** (0.02)	0.18*** (0.01)	0.39*** (0.02)	0.37*** (0.02)
$\beta_{x,0}$	0.59 (0.02)	0.27 (0.02)	0.63 (0.05)	0.97 (0.04)
$b_{i,-1}$	0.30*** (0.03)	0.31*** (0.04)	0.56*** (0.03)	0.60*** (0.02)
Adj R <sup>2</sup>	0.63	0.66	0.66	0.68
SER	0.35	0.34	0.33	0.34
J-stat	0.287835	0.286511	0.291358	0.287604
OvId p-v	0.82	0.82	0.81	0.82
PANEL B: Post-Independence Subsample: 1997:3 - 2001:4 (18 observations)				
Real GDP Filter:	Quadratic	Hodrick-Prescott	Quadratic	Hodrick-Prescott
$b_{0,+2}$ or $b_{0,+3}$	2.56*** (0.13)	0.63* (0.31)	3.01*** (0.17)	3.76*** (0.22)
$\beta_{0,+2}$ or $\beta_{0,+3}$	3.45 (0.17)	1.37 (0.68)	4.75 (0.27)	4.52 (0.26)
$b_{\pi,+3}$	0.79*** (0.07)	0.83*** (0.14)		
$\beta_{\pi,+3}$	1.07 (0.09)	1.83 (0.31)		
$b_{\pi,+2}$			0.31*** (0.10)	0.41 (0.26)
$\beta_{\pi,+2}$			0.50 (0.16)	0.50 (0.29)
$b_{x,0}$	1.41*** (0.06)	1.09*** (0.08)	1.39*** (0.03)	2.07*** (0.08)
$\beta_{x,0}$	1.70 (0.08)	2.39 (0.18)	1.79 (0.06)	2.50 (0.10)
$b_{i,-1}$	0.26*** (0.03)	0.54*** (0.06)	0.37*** (0.02)	0.17*** (0.08)
Adj R <sup>2</sup>	0.90	0.85	0.86	0.75
SER	0.32	0.40	0.38	0.51
J-stat	0.313208	0.286564	0.303636	0.322399
OvId p-v	0.78	0.82	0.79	0.74

TABLE 7. Forward-Looking Taylor Rules: GMM Estimates on RPIX and Real-Time Real GDP Gap

EXPLANATORY NOTE TO TABLE 7: All data are quarterly and for the United Kingdom; inflation is computed using the RPIX and the output gap using real-time real GDP data; the method of estimation is GMM; the instrument set includes 4 lags of all (3) variables in the estimated equation, (4.9), with  $k = 2, 3$ , alternatively, and  $q = 0$ ; standard errors for the directly estimated (reduced-form) coefficients (the  $b$ 's) in parentheses are calculated using a Newey-West weighting matrix robust to error autocorrelation and heteroskedasticity of unknown form; \*\*\*, \*\*, \* = statistical significance at the 1, 5, 10% level, respectively; standard errors for the indirectly estimated (structural-form) coefficients (the  $\beta$ 's) are computed via the delta method (see Appendix B); Adj R<sup>2</sup> = adjusted R<sup>2</sup>; SER = standard error of regression; J stat = J-statistic: equals the minimized value of the objective function in GMM estimation and is used, following Hansen (1982), to test the validity of overidentifying restrictions when there are more instruments than parameters to estimate, like in our case here (we have  $3 \times 4 + 1 = 13$  instruments, including the constant, to estimate 4 parameters, and so there are  $13 - 4 = 9$  overidentifying restrictions: under the null that the overidentifying restrictions are satisfied, the J-statistic times the number of regression observations is distributed asymptotically  $\chi^2(m)$  with degrees of freedom  $m$  equal to the number of overidentifying restrictions, 9 in our case); OvId p-v = probability value of the above-summarized Hansen test for  $m = 9$  overidentifying restrictions.

PANEL A: Pre-Independence Subsample: 1992:4 – 1997:1 (18 observations)				
Real GDP Data:	Final /Revised/		Real-Time /Initial/	
Real GDP Filter:	Quadratic	Hodrick-Prescott	Quadratic	Hodrick-Prescott
$b_{0,+2}$	2.02*** (0.44)	1.22*** (0.12)	1.29*** (0.28)	1.09*** (0.23)
$\beta_{0,+2}$	6.73 (1.46)	3.46 (0.34)	4.18 (0.90)	3.77 (0.81)
$b_{\pi,+2}$	0.08 (0.09)	0.29*** (0.02)	0.25*** (0.04)	0.20*** (0.04)
$\beta_{\pi,+2}$	0.27 (0.30)	0.83 (0.07)	0.80 (0.14)	0.71 (0.13)
$b_{x,0}$	0.47*** (0.05)	0.20*** (0.03)	0.28*** (0.03)	0.30*** (0.03)
$\beta_{x,0}$	0.60 (0.18)	0.57 (0.07)	0.48 (0.08)	1.05 (0.10)
$b_{i,-1}$	0.70*** (0.04)	0.65*** (0.02)	0.69*** (0.03)	0.71*** (0.03)
Adj R <sup>2</sup>	0.64	0.78	0.73	0.72
SER	0.35	0.27	0.30	0.31
J-stat	0.241222	0.284699	0.278124	0.292866
OvId p-v	0.89	0.82	0.83	0.81

PANEL B: Post-Independence Subsample:				
	1997:3 – 2004:4 (28 observations)		1997:3 – 2001:4 (18 observations)	
Real GDP Data:	Final /Revised/		Real-Time /Initial/	
Real GDP Filter:	Quadratic	Hodrick-Prescott	Quadratic	Hodrick-Prescott
$b_{0,+2}$	0.17 (0.16)	−0.13 (0.14)	2.49*** (0.31)	1.70*** (0.21)
$\beta_{0,+2}$	0.61 (0.56)	−1.96 (2.18)	4.33 (0.54)	3.31 (0.41)
$b_{\pi,+2}$	0.48*** (0.03)	0.23*** (0.03)	0.35*** (0.04)	0.47*** (0.03)
$\beta_{\pi,+2}$	1.70 (0.11)	3.52 (0.42)	0.61 (0.06)	0.91 (0.05)
$b_{x,0}$	0.28*** (0.05)	−0.13* (0.07)	0.83*** (0.15)	0.83*** (0.11)
$\beta_{x,0}$	0.74 (0.16)	−2.01 (1.15)	1.51 (0.25)	1.60 (0.21)
$b_{i,-1}$	0.72*** (0.03)	0.93*** (0.02)	0.42*** (0.04)	0.49*** (0.03)
Adj R <sup>2</sup>	0.93	0.91	0.96	0.95
SER	0.33	0.37	0.20	0.24
J-stat	0.248857	0.200585	0.193018	0.285365
OvId p-v	0.64	0.78	0.94	0.82

TABLE 8. Forward-Looking Taylor Rules: GMM Estimates on RPISA

EXPLANATORY NOTE TO TABLE 8: All data are quarterly and for the United Kingdom; inflation is computed using the RPI but *seasonally adjusted*; the method of estimation is GMM; the instrument set includes 4 lags of all (3) variables in the estimated equation, (4.9), with  $k = 2$  and  $q = 0$ ; standard errors for the directly estimated (reduced-form) coefficients (the  $b$ 's) in parentheses are calculated using a Newey-West weighting matrix robust to error autocorrelation and heteroskedasticity of unknown form; \*\*\*, \*\*, \* = statistical significance at the 1, 5, 10% level, respectively; standard errors for the indirectly estimated (structural-form) coefficients (the  $\beta$ 's) are computed via the delta method (see Appendix B); Adj R<sup>2</sup> = adjusted R<sup>2</sup>; SER = standard error of regression; J stat = J-statistic: equals the minimized value of the objective function in GMM estimation and is used, following Hansen (1982), to test the validity of overidentifying restrictions when there are more instruments than parameters to estimate, like in our case here (we have  $3 \times 4 + 1 = 13$  instruments, including the constant, to estimate 4 parameters, and so there are  $13 - 4 = 9$  overidentifying restrictions: under the null that the overidentifying restrictions are satisfied, the J-statistic times the number of regression observations is distributed asymptotically  $\chi^2(m)$  with degrees of freedom  $m$  equal to the number of overidentifying restrictions, 9 in our case); OvId p-v = probability value of the above-summarized Hansen test for  $m = 9$  overidentifying restrictions.

PANEL A: Pre-Independence Subsample: 1992:4 - 1997:1 (18 observations)				
Real GDP Filter:	Quadratic	Hodrick-Prescott	Quadratic	Hodrick-Prescott
$b_{0,+2}$ or $b_{0,+3}$	2.38*** (0.26)	1.69*** (0.14)	2.38*** (0.10)	1.11*** (0.27)
$\beta_{0,+2}$ or $\beta_{0,+3}$	5.84 (0.64)	2.52 (0.21)	5.74 (0.25)	3.33 (0.80)
$b_{\pi,+3}$	0.02 (0.18)	0.84*** (0.11)		
$\beta_{\pi,+3}$	0.06 (0.42)	1.25 (0.16)		
$b_{\pi,+2}$			0.04 (0.07)	0.30*** (0.07)
$\beta_{\pi,+2}$			0.10 (0.17)	0.89 (0.23)
$b_{x,0}$	0.29*** (0.02)	0.20*** (0.01)	0.29*** (0.02)	0.40*** (0.02)
$\beta_{x,0}$	0.58 (0.06)	0.27 (0.02)	0.74 (0.05)	1.21 (0.07)
$b_{i,-1}$	0.59*** (0.04)	0.33*** (0.03)	0.59*** (0.02)	0.67*** (0.03)
Adj R <sup>2</sup>	0.69	0.70	0.70	0.67
SER	0.33	0.32	0.32	0.34
J-stat	0.276950	0.301620	0.277380	0.268906
OvId p-v	0.84	0.80	0.83	0.85
PANEL B: Post-Independence Subsample: 1997:3 - 2001:4 (18 observations)				
Real GDP Filter:	Quadratic	Hodrick-Prescott	Quadratic	Hodrick-Prescott
$b_{0,+2}$ or $b_{0,+3}$	2.41*** (0.12)	2.42*** (0.53)	2.35*** (0.16)	0.12 (0.41)
$\beta_{0,+2}$ or $\beta_{0,+3}$	4.67 (0.23)	7.27 (1.60)	4.58 (0.31)	0.41 (1.42)
$b_{\pi,+3}$	0.29*** (0.06)	-0.17 (0.25)		
$\beta_{\pi,+3}$	0.57 (0.11)	-0.50 (0.75)		
$b_{\pi,+2}$			0.30*** (0.04)	0.67*** (0.17)
$\beta_{\pi,+2}$			0.59 (0.08)	2.32 (0.60)
$b_{x,0}$	1.29*** (0.06)	1.28*** (0.22)	1.28*** (0.08)	0.92*** (0.24)
$\beta_{x,0}$	1.87 (0.12)	3.84 (0.66)	1.83 (0.16)	3.17 (0.82)
$b_{i,-1}$	0.48*** (0.03)	0.67*** (0.08)	0.49*** (0.03)	0.71*** (0.13)
Adj R <sup>2</sup>	0.86	0.64	0.85	0.76
SER	0.38	0.61	0.40	0.50
J-stat	0.309280	0.266308	0.314787	0.294176
OvId p-v	0.78	0.85	0.77	0.81

TABLE 9. Forward-Looking Taylor Rules: GMM Estimates on RPIXSA and Real-Time Real GDP Gap

EXPLANATORY NOTE TO TABLE 9: All data are quarterly and for the United Kingdom; inflation is computed using the RPIX but *seasonally adjusted* and the output gap using *real-time* real GDP data; the method of estimation is GMM; the instrument set includes 4 lags of all (3) variables in the estimated equation, (4.9), with  $k = 2, 3$ , alternatively, and  $q = 0$ ; standard errors for the directly estimated (reduced-form) coefficients (the  $b$ 's) in parentheses are calculated using a Newey-West weighting matrix robust to error autocorrelation and heteroskedasticity of unknown form; \*\*\*, \*\*, \* = statistical significance at the 1, 5, 10% level, respectively; standard errors for the indirectly estimated (structural-form) coefficients (the  $\beta$ 's) are computed via the delta method (see Appendix B); Adj R<sup>2</sup> = adjusted R<sup>2</sup>; SER = standard error of regression; J stat = J-statistic: equals the minimized value of the objective function in GMM estimation and is used, following Hansen (1982), to test the validity of overidentifying restrictions when there are more instruments than parameters to estimate, like in our case here (we have  $3 \times 4 + 1 = 13$  instruments, including the constant, to estimate 4 parameters, and so there are  $13 - 4 = 9$  overidentifying restrictions: under the null that the overidentifying restrictions are satisfied, the J-statistic times the number of regression observations is distributed asymptotically  $\chi^2(m)$  with degrees of freedom  $m$  equal to the number of overidentifying restrictions, 9 in our case); OvId p-v = probability value of the above-summarized Hansen test for  $m = 9$  overidentifying restrictions.



PANEL A: Pre-Independence Subsample: 1992:4 – 1997:1 (18 observations)		
Real GDP Data:	Final /Revised/	Real-Time /Initial/
$b_{0,+2}$	0.81*** (0.14)	1.13*** (0.29)
$\beta_{0,+2}$	1.95 (0.33)	2.53 (0.65)
$b_{\pi,+2}$	0.57*** (0.04)	0.49*** (0.05)
$\beta_{\pi,+2}$	1.36 (0.09)	1.11 (0.12)
$b_{y,0}$	-0.01 (0.02)	0.01 (0.02)
$\beta_{y,0}$	-0.02 (0.04)	0.02 (0.04)
$b_{i,-1}$	0.58*** (0.01)	0.56*** (0.03)
Adj R <sup>2</sup>	0.68	0.67
SER	0.33	0.33
J-stat	0.283959	0.290898
OvId p-v	0.82	0.81
PANEL B: Post-Independence Subsample: 1997:1 – 2004:4 (28 observations)		
Real GDP Data:	Final /Revised/	Real-Time /Initial/
$b_{0,+2}$	-0.08 (0.08)	1.61*** (0.25)
$\beta_{0,+2}$	-0.64 (0.61)	2.88 (0.45)
$b_{\pi,+2}$	0.30*** (0.04)	0.56*** (0.04)
$\beta_{\pi,+2}$	2.27 (0.29)	1.00 (0.08)
$b_{y,0}$	0.01 (0.04)	0.12 (0.07)
$\beta_{y,0}$	0.08 (0.37)	0.21 (0.13)
$b_{i,-1}$	0.87*** (0.01)	0.44*** (0.05)
Adj R <sup>2</sup>	0.93	0.88
SER	0.33	0.36
J-stat	0.199089	0.262429
OvId p-v	0.78	0.86

TABLE 10. Forward-Looking Taylor Rules: GMM Estimates on RPI and Real GDP Growth

EXPLANATORY NOTE TO TABLE 10: All data are quarterly and for the United Kingdom; inflation is computed using the RPI and real GDP *growth* (not gap); the method of estimation is GMM; the instrument set includes 4 lags of all (3) variables in the estimated equation, (4.9), with  $k = 2$  and  $q = 0$ ; standard errors for the directly estimated (reduced-form) coefficients (the  $b$ 's) in parentheses are calculated using a Newey-West weighting matrix robust to error autocorrelation and heteroskedasticity of unknown form; \*\*\*, \*\*, \* = statistical significance at the 1, 5, 10% level, respectively; standard errors for the indirectly estimated (structural-form) coefficients (the  $\beta$ 's) are computed via the delta method (see Appendix B); Adj R<sup>2</sup> = adjusted R<sup>2</sup>; SER = standard error of regression; J stat = J-statistic: equals the minimized value of the objective function in GMM estimation and is used, following Hansen (1982), to test the validity of overidentifying restrictions when there are more instruments than parameters to estimate, like in our case here (we have  $3 \times 4 + 1 = 13$  instruments, including the constant, to estimate 4 parameters, and so there are  $13 - 4 = 9$  overidentifying restrictions: under the null that the overidentifying restrictions are satisfied, the J-statistic times the number of regression observations is distributed asymptotically  $\chi^2(m)$  with degrees of freedom  $m$  equal to the number of overidentifying restrictions, 9 in our case); OvId p-v = probability value of the above-summarized Hansen test for  $m = 9$  overidentifying restrictions.

PANEL A: Pre-Independence Subsample: 1992:4 – 1997:1 (18 observations)		
Real GDP Filter:	Quadratic	Hodrick-Prescott
$b_{0,+2}$	3.04*** (0.02)	1.40*** (0.02)
$b_{\pi,+2}$	0.11*** (0.00)	0.41*** (0.02)
$b_{x,0}$	0.39*** (0.00)	0.25*** (0.01)
$b_{e,0}$	−0.02*** (0.00)	−0.02*** (0.00)
$b_{i,-1}$	0.49*** (0.00)	0.57*** (0.00)
Adj R <sup>2</sup>	0.80	0.75
SER	0.27	0.29
J-stat	0.324132	0.306326
OvId p-v	0.76	0.79
PANEL B: Post-Independence Subsample: 1997:1 – 2004:4 (28 observations)		
Real GDP Filter:	Quadratic	Hodrick-Prescott
$b_{0,+2}$	1.10*** (0.01)	1.16*** (0.09)
$b_{\pi,+2}$	0.02 (0.03)	0.01 (0.02)
$b_{x,0}$	0.06*** (0.01)	0.09*** (0.02)
$b_{e,0}$	0.05*** (0.00)	0.05*** (0.00)
$b_{i,-1}$	0.76*** (0.01)	0.75*** (0.01)
Adj R <sup>2</sup>	0.93	0.93
SER	0.33	0.32
J-stat	0.321897	0.211948
OvId p-v	0.74	0.75

TABLE 11. Forward-Looking Taylor Rules: GMM Estimates on RPI, Real GDP Gap and NEER

EXPLANATORY NOTE TO TABLE 11: All data are quarterly and for the United Kingdom; inflation is computed using the RPI; the method of estimation is GMM; the instrument set includes 4 lags of all (4) variables in the estimated equation, (4.9), with an explicit *nominal effective exchange rate* (NEER) index term,  $k = 2$  and  $q = 0$ ; standard errors for the directly estimated (reduced-form) coefficients (the  $b$ 's) in parentheses are calculated using a Newey-West weighting matrix robust to error autocorrelation and heteroskedasticity of unknown form; \*\*\*, \*\*, \* = statistical significance at the 1, 5, 10% level, respectively; standard errors for the indirectly estimated (structural-form) coefficients (the  $\beta$ 's) were not computed because of the likely misspecification, as discussed in the main text; Adj R<sup>2</sup> = adjusted R<sup>2</sup>; SER = standard error of regression; J stat = J-statistic: equals the minimized value of the objective function in GMM estimation and is used, following Hansen (1982), to test the validity of overidentifying restrictions when there are more instruments than parameters to estimate, like in our case here (we have  $3 \times 4 + 1 = 13$  instruments, including the constant, to estimate 4 parameters, and so there are  $13 - 4 = 9$  overidentifying restrictions: under the null that the overidentifying restrictions are satisfied, the J-statistic times the number of regression observations is distributed asymptotically  $\chi^2(m)$  with degrees of freedom  $m$  equal to the number of overidentifying restrictions, 9 in our case); OvId p-v = probability value of the above-summarized Hansen test for  $m = 9$  overidentifying restrictions.

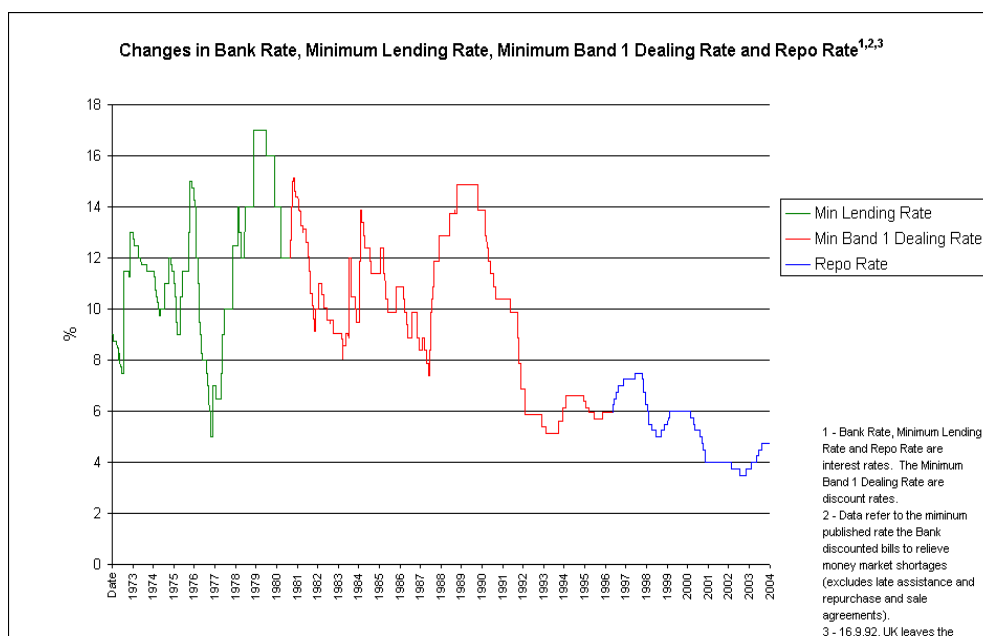


FIGURE 1. UK: Evolution of Bank of England's Operating Instrument and Reference Interest Rate, in % p.a.

GRAPH SOURCE: Bank of England (BoE), website.

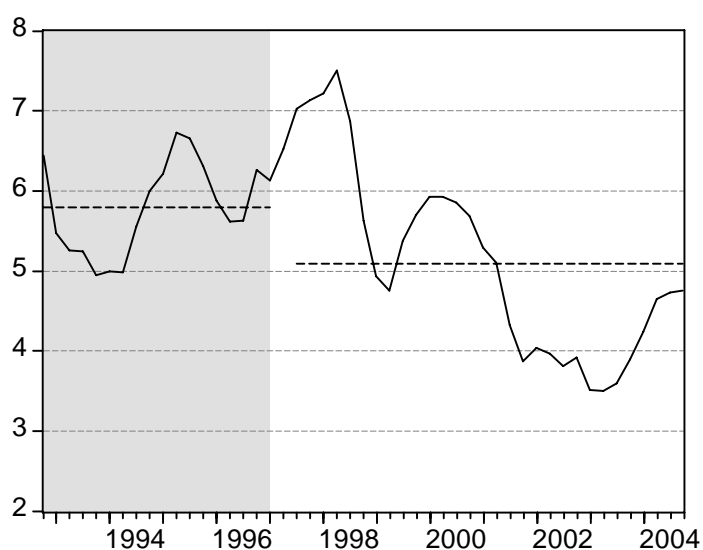


FIGURE 2. UK under Inflation Targeting: 3-Month Treasury Bill Rate, in % p.a., Before and After Bank of England's Operational Independence (the shadowed zone corresponds to the pre-independence subsample, the dashed lines indicate the average interest rate for each subsample)

DATA SOURCE: Office of National Statistics (ONS), website.

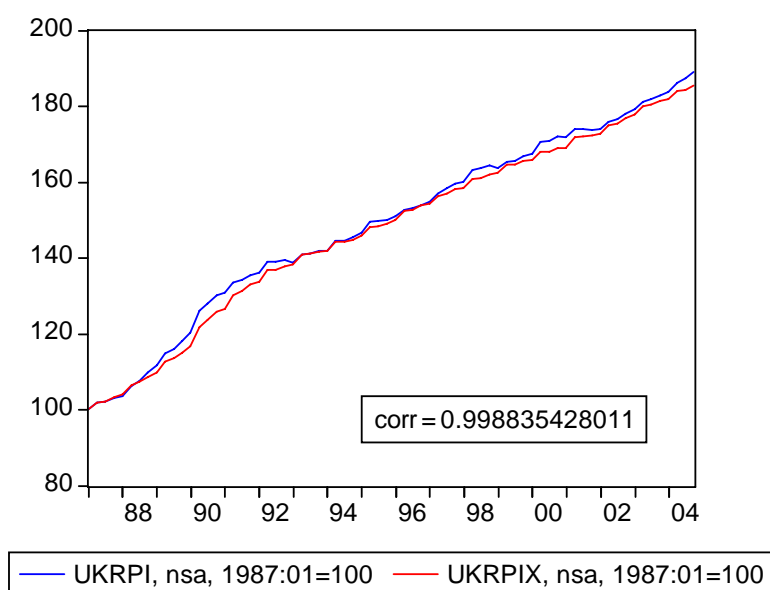


FIGURE 3. UK Price Level Evolution: RPI vs RPIX

DATA SOURCE: Office of National Statistics (ONS), website.

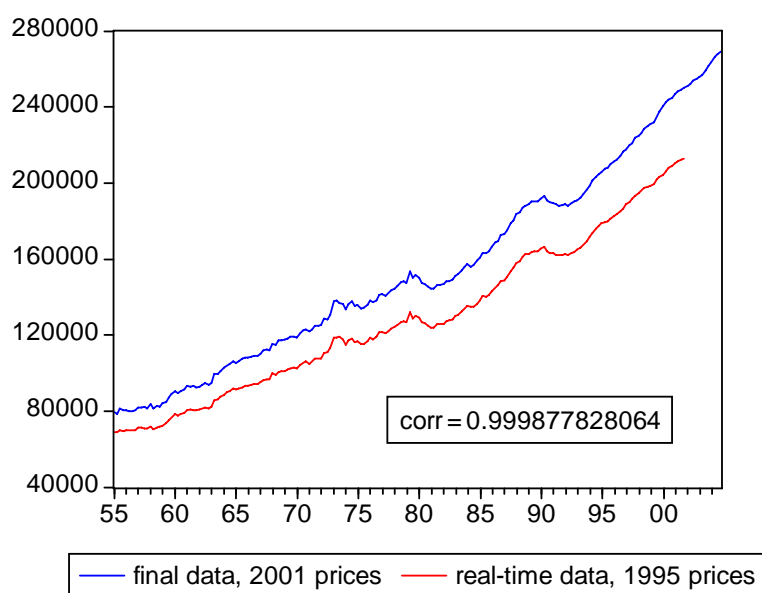


FIGURE 4. UK Output Evolution: Final Real GDP vs Real-Time Real GDP

DATA SOURCE: Office of National Statistics (ONS), website.

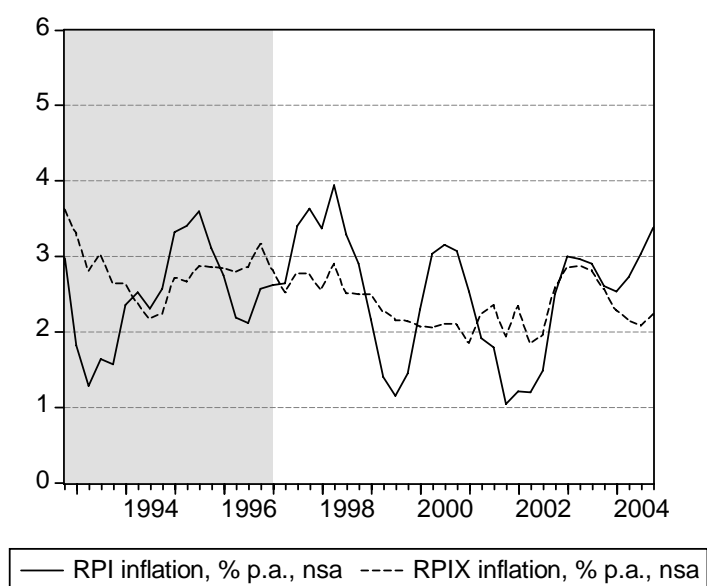


FIGURE 5. UK under Inflation Targeting: RPI Inflation vs RPIX Inflation Before and After Bank of England's Operational Independence (the shadowed zone corresponds to the pre-independence subsample)

DATA SOURCE: Office of National Statistics (ONS), website.

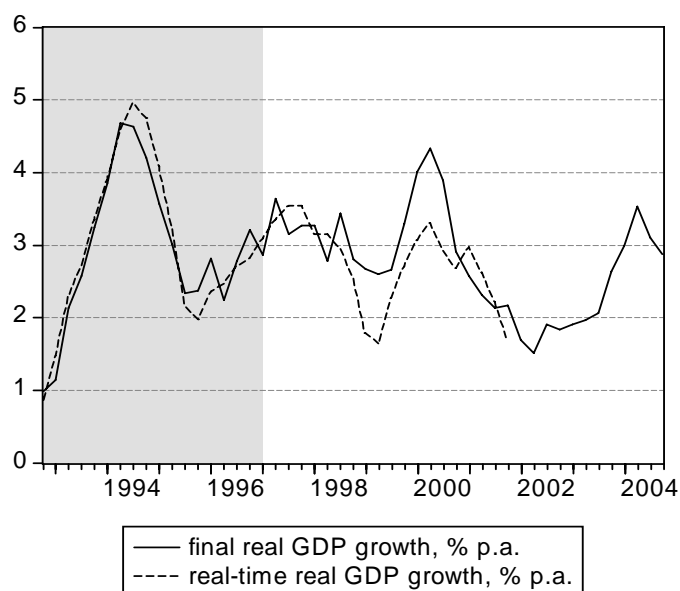


FIGURE 6. UK under Inflation Targeting: Real GDP Growth Before and After Bank of England's Operational Independence, Final vs Real-Time Data (the shadowed zone corresponds to the pre-independence subsample)

DATA SOURCE: Office of National Statistics (ONS), website.

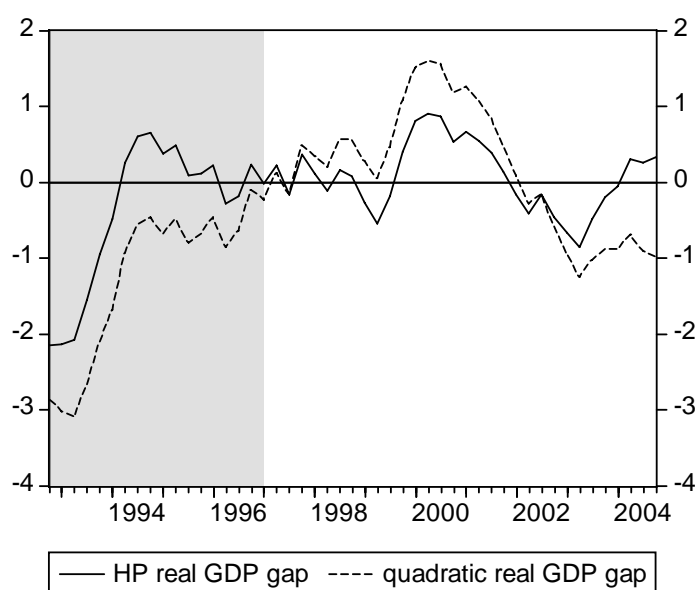


FIGURE 7. UK under Inflation Targeting: Final Real GDP Gap, in % Deviation from Potential, Before and After Bank of England's Operational Independence, Hodrick-Prescott vs Quadratic Filtered Measure (the shadowed zone corresponds to the pre-independence subsample)

DATA SOURCE: Office of National Statistics (ONS), website.

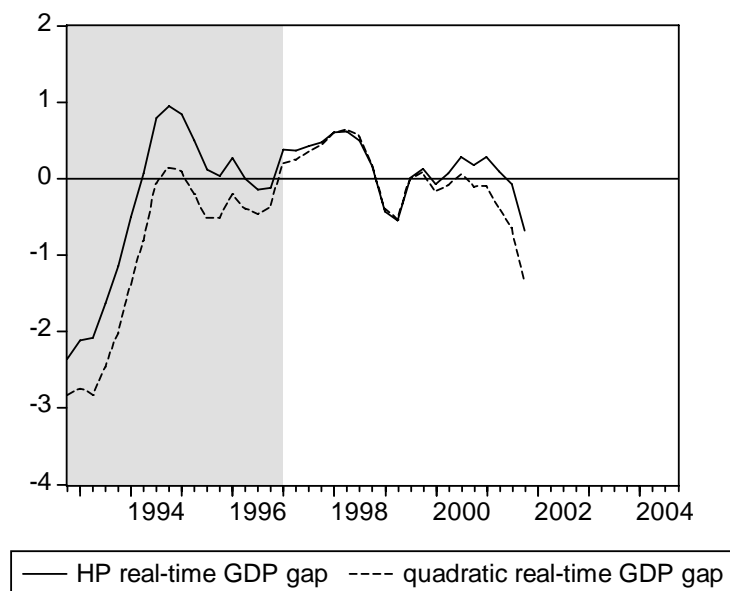


FIGURE 8. UK under Inflation Targeting: Real-Time Real GDP Gap, in % Deviation from Potential, Before and After Bank of England's Operational Independence, Hodrick-Prescott vs Quadratic Filtered Measures (the shadowed zone corresponds to the pre-independence subsample, Nelson-Nikolov (2001) real-time real GDP data for the UK end in 2001:4)

DATA SOURCE: Office of National Statistics (ONS), website.

## 7. APPENDIX B: ANALYTICAL DERIVATION OF DELTA METHOD STANDARD ERRORS

We here explain how we computed standard errors for the parameters of interest in the estimated *forward*- and *backward*-looking Taylor rules for the UK during the inflation targeting period by applying the *delta method*.

**7.1. The Delta Method: Essence and Literature.** The delta method is a technique for *approximating* the moments of *functions* of *random* variables. Oehlert (1992, p. 27) attributes its first fairly rigorous statement, in a less general form, to Cramér (1946, p. 353). It is also used to compute the moments of an approximating *asymptotic distribution*, for example in Rao (1965, p. 319) or Bishop, Feinberg and Holland (1975, p. 48). Hurt (1976), Loève (1977, p. 166 and p. 276) and Lehmann (1983, p. 106, Theorem 5.1, and p. 109, Theorem 5.1b) have extended Cramér's results theoretically, while other authors (see the note by Oehlert (1992)) have provided applications and examples.

In what follows we report in detail how we computed the *standard errors* for all *indirectly* estimated policy response parameters in our *forward*-looking Taylor rule specifications. We then briefly show the analogy with computing the same statistics for the *backward*-looking Taylor rules in our context, which is exploited here for the case of *four* structural parameters to recover.

**7.2. The Delta Method: Application to Forward-Looking Taylor Rules.** As described in section 4 of the main text, in the present paper we focused on estimating forward-looking versions of the Taylor rule in the tradition of Clarida, Galí and Gertler (1997, 1998 a, b, 2000) and the subsequent literature. In particular, we used the GMM approach to estimate equation (4.9). By defining  $\beta_{0,+k} \equiv \beta_1$ ,  $\beta_{\pi,+k} \equiv \beta_2$ ,  $\beta_{x,+q} \equiv \beta_3$  and  $\beta_{i,-1} \equiv \beta_i(L) \equiv \beta_4$ , (4.9) can also be written as

$$(7.1) \quad i_t = (1 - \beta_4) \beta_1 + (1 - \beta_4) \beta_2 \pi_{t+k} + (1 - \beta_4) \beta_3 x_{t+q} + \beta_4 i_{t-1} + \varepsilon_t,$$

with  $\varepsilon_t$  given by (4.10) in the main text.

But what we estimated was, and the coefficients we obtained in result were, in fact

$$(7.2) \quad i_t = b_1 + b_2 \pi_{t+k} + b_3 x_{t+q} + b_4 i_{t-1} + e_t.$$

Following Surico (2004), among others, let us call the *directly estimated* coefficients in (7.2) *reduced-form* parameters and stack them in a vector (as is common in econometrics, we denote below these *estimates* by a *hat*),  $\hat{\mathbf{b}} \equiv \left( \hat{b}_1 \quad \hat{b}_2 \quad \hat{b}_3 \quad \hat{b}_4 \right)'$ .

However, we are interested to *recover* estimates, with standard errors, and interpret what we may call (respective) *structural(-form)* parameters,  $\hat{\beta} \equiv \left( \hat{\beta}_1 \quad \hat{\beta}_2 \quad \hat{\beta}_3 \quad \hat{\beta}_4 \right)'$ .

It is easy to see how both sets of *true* (or *population*) parameters (denoted below by a *zero superscript*, as in classical econometrics), in (7.2) and in (7.1), are related:

$$b_1^0 \equiv (1 - \beta_4^0) \beta_1^0,$$

$$b_2^0 \equiv (1 - \beta_4^0) \beta_2^0,$$

$$b_3^0 \equiv (1 - \beta_4^0) \beta_3^0,$$

$$b_4^0 \equiv \beta_4^0.$$

Hence (inversely),

$$\beta_1^0 \equiv \frac{b_1^0}{1 - \beta_4^0} \equiv \frac{b_1^0}{1 - b_4^0},$$

$$\beta_2^0 \equiv \frac{b_2^0}{1 - \beta_4^0} \equiv \frac{b_2^0}{1 - b_4^0},$$

$$\beta_3^0 \equiv \frac{b_3^0}{1 - \beta_4^0} \equiv \frac{b_3^0}{1 - b_4^0},$$

$$\beta_4^0 \equiv b_4^0.$$

We have thus expressed all true (or population) *structural*-form parameters, in which we are theoretically interested, as respective functions of the true (or population) *reduced*-form parameters, for which we have obtained (direct) empirical *estimates* from our sample. In a more compact notation, this can be written as a vector-valued function

$$\boldsymbol{\beta}^0 = f(\mathbf{b}^0),$$

or, more precisely,

$$\begin{bmatrix} \beta_1^0 \\ \beta_2^0 \\ \beta_3^0 \\ \beta_4^0 \end{bmatrix} = \begin{bmatrix} f(b_1^0, b_4^0) \\ f(b_2^0, b_4^0) \\ f(b_3^0, b_4^0) \\ b_4^0 \end{bmatrix} = \begin{bmatrix} \frac{b_1^0}{1-b_4^0} \\ \frac{b_2^0}{1-b_4^0} \\ \frac{b_3^0}{1-b_4^0} \\ b_4^0 \end{bmatrix}.$$

Having then estimated the sample *reduced*-form parameters  $\hat{\mathbf{b}}$ , we can approximate the sample *structural*-form parameters  $\hat{\boldsymbol{\beta}}$  by

$$\hat{\boldsymbol{\beta}} = f(\hat{\mathbf{b}}),$$

or,

$$\begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \hat{\beta}_3 \\ \hat{\beta}_4 \end{bmatrix} = \begin{bmatrix} f(\hat{b}_1, \hat{b}_4) \\ f(\hat{b}_2, \hat{b}_4) \\ f(\hat{b}_3, \hat{b}_4) \\ \hat{b}_4 \end{bmatrix} = \begin{bmatrix} \frac{\hat{b}_1}{1-\hat{b}_4} \\ \frac{\hat{b}_2}{1-\hat{b}_4} \\ \frac{\hat{b}_3}{1-\hat{b}_4} \\ \hat{b}_4 \end{bmatrix}.$$

To compute the (approximate) *standard errors* for the (indirect) sample estimates of the structural parameters  $\hat{\boldsymbol{\beta}}$ , we need to recur to the now standard technique in similar circumstances known as the *delta method*.

**7.2.1. Taylor Approximation to Nonlinear Function of Random Variables (Sample Estimators).** The first step in it is to apply *Taylor expansion* to approximate the *nonlinear* relationship among the respective *estimated* parameters around their *true* values. Note that the Taylor approximation in this case is an approximation to a *function* of a random variable, the random variable being the *estimator* for each regressor  $i$ .

**Scalar-Valued Function.** Let us begin, for clarity, by considering the *scalar* case:

$$\beta_i^0 \equiv \frac{b_i^0}{1-b_4^0}, \quad i = 1, 2, 3.$$

An assumption is made that the (sample) *estimators*  $\hat{\beta}_i$  for each respective regressor  $i$  are (asymptotically) *normally* distributed around their respective true (or population) values  $\beta_i^0$  with variance  $(\omega_{\beta_i}^0)^2$ :

$$(7.3) \quad \hat{\beta}_i \sim \mathcal{N}\left(\beta_i^0, (\omega_{\beta_i}^0)^2\right) \Leftrightarrow (\hat{\beta}_i - \beta_i^0) \sim \mathcal{N}\left(0, (\omega_{\beta_i}^0)^2\right)$$

The *first-order* (or *linear*) Taylor approximation around the *true* value  $\beta_i^0$  of the *estimated* parameter of interest  $\hat{\beta}_i$  is then given by:<sup>24</sup>

$$f(\hat{\beta}_i) \approx f(\beta_i^0) + f'(\beta_i^0)(\hat{\beta}_i - \beta_i^0),$$

or

$$(7.4) \quad f(\hat{\beta}_i) - f(\beta_i^0) \approx f'(\beta_i^0)(\hat{\beta}_i - \beta_i^0).$$

Taking expectations of both sides of (7.4):

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<sup>24</sup>See, for instance, Green (2000), p. 50.



$$E \left[ f \left( \widehat{\beta}_i \right) - f \left( \beta_i^0 \right) \right] \approx \underbrace{f' \left( \beta_i^0 \right)}_{=const} \underbrace{E \left( \widehat{\beta}_i - \beta_i^0 \right)}_{=E \left[ \widehat{\beta}_i \right] - \beta_i^0} = 0.$$

So, rewriting for the *mean* of the function of the estimator  $f \left( \widehat{\beta}_i \right)$ :

$$(7.5) \quad E \left[ f \left( \widehat{\beta}_i \right) \right] = f \left( \beta_i^0 \right),$$

and the *estimated* (or *feasible*) equivalent to (7.5) will thus be:

$$(7.6) \quad \widehat{E} \left[ f \left( \widehat{\beta}_i \right) \right] = f \left( \widehat{\beta}_i \right).$$

Now, squaring both sides of (7.4):

$$\left[ f \left( \widehat{\beta}_i \right) - f \left( \beta_i^0 \right) \right]^2 \approx \left[ f' \left( \beta_i^0 \right) \left( \widehat{\beta}_i - \beta_i^0 \right) \right]^2,$$

the RHS of which can also be written as

$$\left[ f \left( \widehat{\beta}_i \right) - f \left( \beta_i^0 \right) \right]^2 \approx \left[ f' \left( \beta_i^0 \right) \right]^2 \left( \widehat{\beta}_i - \beta_i^0 \right)^2,$$

and, again, taking expectations from the squared expression just above:

$$\underbrace{E \left[ f \left( \widehat{\beta}_i \right) - f \left( \beta_i^0 \right) \right]^2}_{\equiv Var[f(\widehat{\beta}_i)]} \approx \underbrace{\left[ f' \left( \beta_i^0 \right) \right]^2}_{=const} \underbrace{E \left[ \left( \widehat{\beta}_i - \beta_i^0 \right)^2 \right]}_{\equiv Var[\widehat{\beta}_i] \equiv (\omega_{\beta_i}^0)^2 \text{ from (7.3)}}.$$

So, re-writing for the *variance* of the function of the estimator  $f \left( \widehat{\beta}_i \right)$ :

$$(7.7) \quad Var \left[ f \left( \widehat{\beta}_i \right) \right] = \left[ f' \left( \beta_i^0 \right) \right]^2 \left( \omega_{\beta_i}^0 \right)^2,$$

and the *estimated* (or *feasible*) equivalent to (7.7) will thus be

$$(7.8) \quad \widehat{Var} \left[ f \left( \widehat{\beta}_i \right) \right] = \left[ f' \left( \widehat{\beta}_i \right) \right]^2 \left( \widehat{\omega}_{\beta_i} \right)^2.$$

For the distribution of  $f \left( \widehat{\beta}_i \right)$ , a *function* of the parameter *estimator* for each *regressor*  $i$  (in the given *sample* of size  $T$ ), we have thus verified that

$$f \left( \widehat{\beta}_i \right) \overset{as}{\sim} \mathcal{N} \left( f \left( \beta_i^0 \right), \left[ f' \left( \beta_i^0 \right) \right]^2 \left( \omega_{\beta_i}^0 \right)^2 \right).$$

We have also established, in effect, that the distribution of  $f \left( \widehat{\beta}_i \right) - f \left( \beta_i^0 \right)$ , a *function* of the parameter *estimator* for each *regressor*  $i$  (in the given *sample* of size  $T$ ) expressed in *deviation* from the same function of its *true* (or *population*) mean, converges *almost surely* to the *normal* distribution with *zero* mean and variance equal to  $\left[ f' \left( \beta_i^0 \right) \right]^2 \left( \omega_{\beta_i}^0 \right)^2$ :

$$(7.9) \quad f \left( \widehat{\beta}_i \right) - f \left( \beta_i^0 \right) \overset{as}{\sim} \mathcal{N} \left( 0, \left[ f' \left( \beta_i^0 \right) \right]^2 \left( \omega_{\beta_i}^0 \right)^2 \right).$$

Analogously, one can show that

$$f \left( \widehat{b}_i \right) - f \left( b_i^0 \right) \overset{as}{\sim} \mathcal{N} \left( 0, \left[ f' \left( b_i^0 \right) \right]^2 \left( \sigma_{b_i}^0 \right)^2 \right),$$

where  $\left( \sigma_{b_i}^0 \right)^2 \equiv Var \left[ \widehat{b}_i \right] \equiv E \left[ \left( \widehat{b}_i - b_i^0 \right)^2 \right]$ .

Now, in our case here the *specific* functional relation between the structural parameters (to be recovered) and the reduced-form ones (that were estimated) is given by  $\beta_4 \equiv b_4$  and

$$\beta_i \equiv f(b_i, b_4) \equiv \left( \frac{b_i}{1-b_4} \right), \quad i = 1, 2, 3.$$

Hence, the *first* derivative w.r.t.  $b_1$  will be

$$f'(\beta_1) \equiv \frac{\partial f\left(\frac{b_1}{1-b_4}\right)}{\partial b_1} = \frac{\overbrace{(b_1)'}^{=1} (1-b_4) + b_1 \overbrace{(1-b_4)'}^{=0}}{(1-b_4)^2} = \frac{(1-b_4)}{(1-b_4)^2} = \frac{1}{1-b_4}.$$

The *same* expressions for the first and second derivatives w.r.t.  $b_2$  and  $b_3$  can be analogously obtained. W.r.t.  $b_4$  the *first* derivative will be

$$f'(\beta_4) \equiv \frac{\partial f\left(\frac{b_4}{1-b_4}\right)}{\partial b_4} = \frac{\overbrace{(b_4)'}^{=1} (1-b_4) + b_4 \overbrace{(1-b_4)'}^{=-1}}{(1-b_4)^2} = \frac{(1-b_4) - b_4}{(1-b_4)^2} = \frac{1-2b_4}{(1-b_4)^2}.$$

Vector-Valued Function. Until now we looked at the scalar case, that is, at the case of approximating a function of a *single* random variable (such as any of our sample *estimators*  $i$  taken *individually*), or of approximating a *scalar-valued* function. Generalization to the vector case – that is, the case of approximating a function of *more than one* random variables (such as our sample *estimators*  $i$  taken *altogether* in a vector), or of approximating a *vector-valued* function – builds upon the analogy. We now write, introducing the following matrix notation for the coefficients of the *structural* form:

$$\begin{aligned} \underset{(4 \times 1)}{\hat{\boldsymbol{\beta}}} &\sim \mathcal{N}\left(\underset{(4 \times 1)}{\boldsymbol{\beta}^0}, \underset{(4 \times 4)}{\boldsymbol{\Omega}^0}\right) \Leftrightarrow \left(\underset{(4 \times 1)}{\hat{\boldsymbol{\beta}}} - \underset{(4 \times 1)}{\boldsymbol{\beta}^0}\right) \sim \mathcal{N}\left(\underset{(4 \times 1)}{\mathbf{0}}, \underset{(4 \times 4)}{\boldsymbol{\Omega}^0}\right), \text{ with} \\ \underset{(4 \times 1)}{\hat{\boldsymbol{\beta}}} &\equiv \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \hat{\beta}_3 \\ \hat{\beta}_4 \end{bmatrix}, \quad \underset{(4 \times 1)}{\boldsymbol{\beta}^0} \equiv \begin{bmatrix} \beta_1^0 \\ \beta_2^0 \\ \beta_3^0 \\ \beta_4^0 \end{bmatrix}, \text{ and} \end{aligned}$$

with – in our case of four regressors (including the constant, or intercept) – variance-covariance matrices

$$\begin{aligned} \underset{(4 \times 4)}{\boldsymbol{\Omega}^0} &\equiv \begin{bmatrix} (\omega_{11}^0)^2 & \omega_{12}^0 & \omega_{13}^0 & \omega_{14}^0 \\ \omega_{21}^0 & (\omega_{22}^0)^2 & \omega_{23}^0 & \omega_{24}^0 \\ \omega_{31}^0 & \omega_{32}^0 & (\omega_{33}^0)^2 & \omega_{34}^0 \\ \omega_{41}^0 & \omega_{42}^0 & \omega_{43}^0 & (\omega_{44}^0)^2 \end{bmatrix} \equiv \\ &\equiv \begin{bmatrix} \text{Var}(\beta_1^0) & \text{Cov}(\beta_1^0, \beta_2^0) & \text{Cov}(\beta_1^0, \beta_3^0) & \text{Cov}(\beta_1^0, \beta_4^0) \\ \text{Cov}(\beta_2^0, \beta_1^0) & \text{Var}(\beta_2^0) & \text{Cov}(\beta_2^0, \beta_3^0) & \text{Cov}(\beta_2^0, \beta_4^0) \\ \text{Cov}(\beta_3^0, \beta_1^0) & \text{Cov}(\beta_3^0, \beta_2^0) & \text{Var}(\beta_3^0) & \text{Cov}(\beta_3^0, \beta_4^0) \\ \text{Cov}(\beta_4^0, \beta_1^0) & \text{Cov}(\beta_4^0, \beta_2^0) & \text{Cov}(\beta_4^0, \beta_3^0) & \text{Var}(\beta_4^0) \end{bmatrix} \cong \\ &\cong \begin{bmatrix} \widehat{\text{Var}}(\hat{\beta}_1) & \widehat{\text{Cov}}(\hat{\beta}_1, \hat{\beta}_2) & \widehat{\text{Cov}}(\hat{\beta}_1, \hat{\beta}_3) & \widehat{\text{Cov}}(\hat{\beta}_1, \hat{\beta}_4) \\ \widehat{\text{Cov}}(\hat{\beta}_2, \hat{\beta}_1) & \widehat{\text{Var}}(\hat{\beta}_2) & \widehat{\text{Cov}}(\hat{\beta}_2, \hat{\beta}_3) & \widehat{\text{Cov}}(\hat{\beta}_2, \hat{\beta}_4) \\ \widehat{\text{Cov}}(\hat{\beta}_3, \hat{\beta}_1) & \widehat{\text{Cov}}(\hat{\beta}_3, \hat{\beta}_2) & \widehat{\text{Var}}(\hat{\beta}_3) & \widehat{\text{Cov}}(\hat{\beta}_3, \hat{\beta}_4) \\ \widehat{\text{Cov}}(\hat{\beta}_4, \hat{\beta}_1) & \widehat{\text{Cov}}(\hat{\beta}_4, \hat{\beta}_2) & \widehat{\text{Cov}}(\hat{\beta}_4, \hat{\beta}_3) & \widehat{\text{Var}}(\hat{\beta}_4) \end{bmatrix} \equiv \\ &\equiv \begin{bmatrix} (\hat{\omega}_{11})^2 & \hat{\omega}_{12} & \hat{\omega}_{13} & \hat{\omega}_{14} \\ \hat{\omega}_{21} & (\hat{\omega}_{22})^2 & \hat{\omega}_{23} & \hat{\omega}_{24} \\ \hat{\omega}_{31} & \hat{\omega}_{32} & (\hat{\omega}_{33})^2 & \hat{\omega}_{34} \\ \hat{\omega}_{41} & \hat{\omega}_{42} & \hat{\omega}_{43} & (\hat{\omega}_{44})^2 \end{bmatrix} \equiv \underset{(4 \times 4)}{\hat{\boldsymbol{\Omega}}}. \end{aligned}$$

Analogously, we introduce the following matrix notation, now for the directly estimated coefficients of the *reduced* form:

$$\underset{(4 \times 1)}{\hat{\mathbf{b}}} \sim \mathcal{N}\left(\underset{(4 \times 1)}{\mathbf{b}^0}, \underset{(4 \times 4)}{\mathbf{S}^0}\right) \Leftrightarrow \left(\underset{(4 \times 1)}{\hat{\mathbf{b}}} - \underset{(4 \times 1)}{\mathbf{b}^0}\right) \sim \mathcal{N}\left(\underset{(4 \times 1)}{\mathbf{0}}, \underset{(4 \times 4)}{\mathbf{S}^0}\right), \text{ with}$$

$$\begin{aligned}
\hat{\mathbf{b}}_{(4 \times 1)} &\equiv \begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \hat{b}_3 \\ \hat{b}_4 \end{bmatrix}, \quad \mathbf{b}^0_{(4 \times 1)} \equiv \begin{bmatrix} b_1^0 \\ b_2^0 \\ b_3^0 \\ b_4^0 \end{bmatrix}, \text{ and} \\
\mathbf{S}^0_{(4 \times 4)} &\equiv \begin{bmatrix} (\sigma_{11}^0)^2 & \sigma_{12}^0 & \sigma_{13}^0 & \sigma_{14}^0 \\ \sigma_{21}^0 & (\sigma_{22}^0)^2 & \sigma_{23}^0 & \sigma_{24}^0 \\ \sigma_{31}^0 & \sigma_{32}^0 & (\sigma_{33}^0)^2 & \sigma_{34}^0 \\ \sigma_{41}^0 & \sigma_{42}^0 & \sigma_{43}^0 & (\sigma_{44}^0)^2 \end{bmatrix} \equiv \\
&\equiv \begin{bmatrix} \text{Var}(b_1^0) & \text{Cov}(b_1^0, b_2^0) & \text{Cov}(b_1^0, b_3^0) & \text{Cov}(b_1^0, b_4^0) \\ \text{Cov}(b_2^0, b_1^0) & \text{Var}(b_2^0) & \text{Cov}(b_2^0, b_3^0) & \text{Cov}(b_2^0, b_4^0) \\ \text{Cov}(b_3^0, b_1^0) & \text{Cov}(b_3^0, b_2^0) & \text{Var}(b_3^0) & \text{Cov}(b_3^0, b_4^0) \\ \text{Cov}(b_4^0, b_1^0) & \text{Cov}(b_4^0, b_2^0) & \text{Cov}(b_4^0, b_3^0) & \text{Var}(b_4^0) \end{bmatrix} \cong \\
&\cong \begin{bmatrix} \widehat{\text{Var}}(\hat{b}_1) & \widehat{\text{Cov}}(\hat{b}_1, \hat{b}_2) & \widehat{\text{Cov}}(\hat{b}_1, \hat{b}_3) & \widehat{\text{Cov}}(\hat{b}_1, \hat{b}_4) \\ \widehat{\text{Cov}}(\hat{b}_2, \hat{b}_1) & \widehat{\text{Var}}(\hat{b}_2) & \widehat{\text{Cov}}(\hat{b}_2, \hat{b}_3) & \widehat{\text{Cov}}(\hat{b}_2, \hat{b}_4) \\ \widehat{\text{Cov}}(\hat{b}_3, \hat{b}_1) & \widehat{\text{Cov}}(\hat{b}_3, \hat{b}_2) & \widehat{\text{Var}}(\hat{b}_3) & \widehat{\text{Cov}}(\hat{b}_3, \hat{b}_4) \\ \widehat{\text{Cov}}(\hat{b}_4, \hat{b}_1) & \widehat{\text{Cov}}(\hat{b}_4, \hat{b}_2) & \widehat{\text{Cov}}(\hat{b}_4, \hat{b}_3) & \widehat{\text{Var}}(\hat{b}_4) \end{bmatrix} \equiv \\
&\equiv \begin{bmatrix} (\hat{\sigma}_{11})^2 & \hat{\sigma}_{12} & \hat{\sigma}_{13} & \hat{\sigma}_{14} \\ \hat{\sigma}_{21} & (\hat{\sigma}_{22})^2 & \hat{\sigma}_{23} & \hat{\sigma}_{24} \\ \hat{\sigma}_{31} & \hat{\sigma}_{32} & (\hat{\sigma}_{33})^2 & \hat{\sigma}_{34} \\ \hat{\sigma}_{41} & \hat{\sigma}_{42} & \hat{\sigma}_{43} & (\hat{\sigma}_{44})^2 \end{bmatrix} \equiv \hat{\mathbf{S}}_{(4 \times 4)}.
\end{aligned}$$

In *general*, for any *vector*-valued function of random variables (here, sample estimators)  $\hat{\boldsymbol{\beta}}$  and  $\hat{\mathbf{b}}$ , expanding around the respective true (or population) parameters by applying a first-order (linear) Taylor approximation would result in (note the analogy with the scalar case presented earlier):

$$\begin{aligned}
\begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \hat{\beta}_3 \\ \hat{\beta}_4 \end{bmatrix} &\cong f \begin{bmatrix} \beta_1^0 \\ \beta_2^0 \\ \beta_3^0 \\ \beta_4^0 \end{bmatrix} + \frac{\partial f \begin{bmatrix} \beta_1^0 \\ \beta_2^0 \\ \beta_3^0 \\ \beta_4^0 \end{bmatrix}}{\partial [\beta_1 \ \beta_2 \ \beta_3 \ \beta_4]} \begin{bmatrix} \hat{\beta}_1 - \beta_1^0 \\ \hat{\beta}_2 - \beta_2^0 \\ \hat{\beta}_3 - \beta_3^0 \\ \hat{\beta}_4 - \beta_4^0 \end{bmatrix} \Rightarrow \\
\Rightarrow \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \hat{\beta}_3 \\ \hat{\beta}_4 \end{bmatrix} &\cong \mathcal{N} \left( f \begin{bmatrix} \beta_1^0 \\ \beta_2^0 \\ \beta_3^0 \\ \beta_4^0 \end{bmatrix}, \frac{\partial f \begin{bmatrix} \beta_1^0 \\ \beta_2^0 \\ \beta_3^0 \\ \beta_4^0 \end{bmatrix}}{\partial [\beta_1 \ \beta_2 \ \beta_3 \ \beta_4]} \boldsymbol{\Omega}^0_{(4 \times 4)} \frac{\partial f \begin{bmatrix} \beta_1^0 \\ \beta_2^0 \\ \beta_3^0 \\ \beta_4^0 \end{bmatrix}}{\partial \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix}} \right)
\end{aligned}$$

and

$$\begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \hat{b}_3 \\ \hat{b}_4 \end{bmatrix} \cong f \begin{bmatrix} b_1^0 \\ b_2^0 \\ b_3^0 \\ b_4^0 \end{bmatrix} + \frac{\partial f \begin{bmatrix} b_1^0 \\ b_2^0 \\ b_3^0 \\ b_4^0 \end{bmatrix}}{\partial [b_1 \ b_2 \ b_3 \ b_4]} \begin{bmatrix} \hat{b}_1 - b_1^0 \\ \hat{b}_2 - b_2^0 \\ \hat{b}_3 - b_3^0 \\ \hat{b}_4 - b_4^0 \end{bmatrix} \Rightarrow$$

$$\Rightarrow \begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \hat{b}_3 \\ \hat{b}_4 \end{bmatrix} \cong \mathcal{N} \left( f \begin{bmatrix} b_1^0 \\ b_2^0 \\ b_3^0 \\ b_4^0 \end{bmatrix}, \frac{\partial f \begin{bmatrix} b_1^0 \\ b_2^0 \\ b_3^0 \\ b_4^0 \end{bmatrix}}{\partial \begin{bmatrix} b_1 & b_2 & b_3 & b_4 \end{bmatrix}_{(4 \times 4)}} \mathbf{S}^0 \frac{\partial f \begin{bmatrix} b_1^0 \\ b_2^0 \\ b_3^0 \\ b_4^0 \end{bmatrix}}{\partial \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}} \right).$$

7.2.2. *Analytical Derivation of Our Delta Method Standard Error Computations.* In our special case of functional relationship of  $\beta^0 = f(\mathbf{b}^0)$  estimated by  $\hat{\beta} = f(\hat{\mathbf{b}})$ , we have:

$$\begin{aligned} \hat{\beta} = f(\hat{\mathbf{b}}) &\Leftrightarrow \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \hat{\beta}_3 \\ \hat{\beta}_4 \end{bmatrix} = f \begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \hat{b}_3 \\ \hat{b}_4 \end{bmatrix} = \begin{bmatrix} \frac{\hat{b}_1}{1-\hat{b}_4} \\ \frac{\hat{b}_2}{1-\hat{b}_4} \\ \frac{\hat{b}_3}{1-\hat{b}_4} \\ \hat{b}_4 \end{bmatrix}, \text{ hence} \\ \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \hat{\beta}_3 \\ \hat{\beta}_4 \end{bmatrix} &\cong \begin{bmatrix} \frac{\hat{b}_1}{1-\hat{b}_4} \\ \frac{\hat{b}_2}{1-\hat{b}_4} \\ \frac{\hat{b}_3}{1-\hat{b}_4} \\ \hat{b}_4 \end{bmatrix} + \frac{\partial \begin{bmatrix} \frac{\hat{b}_1}{1-\hat{b}_4} \\ \frac{\hat{b}_2}{1-\hat{b}_4} \\ \frac{\hat{b}_3}{1-\hat{b}_4} \\ \hat{b}_4 \end{bmatrix}}{\partial \begin{bmatrix} b_1 & b_2 & b_3 & b_4 \end{bmatrix}} \begin{bmatrix} \hat{b}_1 - b_1^0 \\ \hat{b}_2 - b_2^0 \\ \hat{b}_3 - b_3^0 \\ \hat{b}_4 - b_4^0 \end{bmatrix} \Rightarrow \\ \Rightarrow \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \hat{\beta}_3 \\ \hat{\beta}_4 \end{bmatrix} &\cong \mathcal{N} \left( \begin{bmatrix} \frac{\hat{b}_1}{1-\hat{b}_4} \\ \frac{\hat{b}_2}{1-\hat{b}_4} \\ \frac{\hat{b}_3}{1-\hat{b}_4} \\ \hat{b}_4 \end{bmatrix}, \frac{\partial \begin{bmatrix} \frac{\hat{b}_1}{1-\hat{b}_4} \\ \frac{\hat{b}_2}{1-\hat{b}_4} \\ \frac{\hat{b}_3}{1-\hat{b}_4} \\ \hat{b}_4 \end{bmatrix}}{\partial \begin{bmatrix} b_1 & b_2 & b_3 & b_4 \end{bmatrix}_{(4 \times 4)}} \hat{\mathbf{S}} \frac{\partial \begin{bmatrix} \frac{\hat{b}_1}{1-\hat{b}_4} \\ \frac{\hat{b}_2}{1-\hat{b}_4} \\ \frac{\hat{b}_3}{1-\hat{b}_4} \\ \hat{b}_4 \end{bmatrix}}{\partial \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}} \right). \end{aligned}$$

Hence:

$$\begin{aligned} \frac{\partial \begin{bmatrix} \frac{\hat{b}_1}{1-\hat{b}_4} \\ \frac{\hat{b}_2}{1-\hat{b}_4} \\ \frac{\hat{b}_3}{1-\hat{b}_4} \\ \hat{b}_4 \end{bmatrix}}{\partial \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}} &= \begin{bmatrix} \frac{1}{1-\hat{b}_4} & 0 & 0 & \frac{1-2\hat{b}_4}{(1-\hat{b}_4)^2} \\ 0 & \frac{1}{1-\hat{b}_4} & 0 & \frac{1-2\hat{b}_4}{(1-\hat{b}_4)^2} \\ 0 & 0 & \frac{1}{1-\hat{b}_4} & \frac{1-2\hat{b}_4}{(1-\hat{b}_4)^2} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \\ &= \frac{1}{(1-\hat{b}_4)^2} \begin{bmatrix} 1-\hat{b}_4 & 0 & 0 & 1-2\hat{b}_4 \\ 0 & 1-\hat{b}_4 & 0 & 1-2\hat{b}_4 \\ 0 & 0 & 1-\hat{b}_4 & 1-2\hat{b}_4 \\ 0 & 0 & 0 & (1-\hat{b}_4)^2 \end{bmatrix}. \end{aligned}$$

Next, the *transpose* of the above matrix will be:

$$\begin{aligned}
& \begin{bmatrix} \frac{1}{1-\hat{b}_4} & 0 & 0 & \frac{1-2\hat{b}_4}{(1-\hat{b}_4)^2} \\ 0 & \frac{1}{1-\hat{b}_4} & 0 & \frac{1-2\hat{b}_4}{(1-\hat{b}_4)^2} \\ 0 & 0 & \frac{1}{1-\hat{b}_4} & \frac{1-2\hat{b}_4}{(1-\hat{b}_4)^2} \\ 0 & 0 & 0 & 1 \end{bmatrix}^T = \\
& = \begin{bmatrix} \frac{1}{1-\hat{b}_4} & 0 & 0 & 0 \\ 0 & \frac{1}{1-\hat{b}_4} & 0 & 0 \\ 0 & 0 & \frac{1}{1-\hat{b}_4} & 0 \\ \frac{1-2\hat{b}_4}{(1-\hat{b}_4)^2} & \frac{1-2\hat{b}_4}{(1-\hat{b}_4)^2} & \frac{1-2\hat{b}_4}{(1-\hat{b}_4)^2} & 1 \end{bmatrix} \partial \begin{bmatrix} \frac{\hat{b}_1}{1-\hat{b}_4} \\ \frac{\hat{b}_2}{1-\hat{b}_4} \\ \frac{\hat{b}_3}{1-\hat{b}_4} \\ \hat{b}_4 \end{bmatrix} = \frac{\partial [b_1 \ b_2 \ b_3 \ b_4]}{\partial [b_1 \ b_2 \ b_3 \ b_4]} = \\
& = \frac{1}{(1-\hat{b}_4)^2} \begin{bmatrix} 1-\hat{b}_4 & 0 & 0 & 0 \\ 0 & 1-\hat{b}_4 & 0 & 0 \\ 0 & 0 & 1-\hat{b}_4 & 0 \\ 1-2\hat{b}_4 & 1-2\hat{b}_4 & 1-2\hat{b}_4 & (1-\hat{b}_4)^2 \end{bmatrix}.
\end{aligned}$$

Now we are ready to write:

$$\begin{aligned}
& \widehat{Var} \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \hat{\beta}_3 \\ \hat{\beta}_4 \end{bmatrix} \cong \widehat{Var} \begin{bmatrix} \frac{\hat{b}_1}{1-\hat{b}_4} \\ \frac{\hat{b}_2}{1-\hat{b}_4} \\ \frac{\hat{b}_3}{1-\hat{b}_4} \\ \hat{b}_4 \end{bmatrix} = \\
& = \frac{1}{(1-\hat{b}_4)^4} \begin{bmatrix} 1-\hat{b}_4 & 0 & 0 & 0 \\ 0 & 1-\hat{b}_4 & 0 & 0 \\ 0 & 0 & 1-\hat{b}_4 & 0 \\ 1-2\hat{b}_4 & 1-2\hat{b}_4 & 1-2\hat{b}_4 & (1-\hat{b}_4)^2 \end{bmatrix} \begin{matrix} \mathbf{S}_{(4 \times 4)} \\ \begin{bmatrix} 1-\hat{b}_4 & 0 & 0 & 1-2\hat{b}_4 \\ 0 & 1-\hat{b}_4 & 0 & 1-2\hat{b}_4 \\ 0 & 0 & 1-\hat{b}_4 & 1-2\hat{b}_4 \\ 0 & 0 & 0 & (1-\hat{b}_4)^2 \end{bmatrix} \end{matrix} = \\
& = \frac{1}{(1-\hat{b}_4)^4} \begin{bmatrix} 1-\hat{b}_4 & 0 & 0 & 0 \\ 0 & 1-\hat{b}_4 & 0 & 0 \\ 0 & 0 & 1-\hat{b}_4 & 0 \\ 1-2\hat{b}_4 & 1-2\hat{b}_4 & 1-2\hat{b}_4 & (1-\hat{b}_4)^2 \end{bmatrix} \begin{bmatrix} (\hat{\sigma}_{11})^2 & \hat{\sigma}_{12} & \hat{\sigma}_{13} & \hat{\sigma}_{14} \\ \hat{\sigma}_{21} & (\hat{\sigma}_{22})^2 & \hat{\sigma}_{23} & \hat{\sigma}_{24} \\ \hat{\sigma}_{31} & \hat{\sigma}_{32} & (\hat{\sigma}_{33})^2 & \hat{\sigma}_{34} \\ \hat{\sigma}_{41} & \hat{\sigma}_{42} & \hat{\sigma}_{43} & (\hat{\sigma}_{44})^2 \end{bmatrix} \times \\
& \times \begin{bmatrix} 1-\hat{b}_4 & 0 & 0 & 1-2\hat{b}_4 \\ 0 & 1-\hat{b}_4 & 0 & 1-2\hat{b}_4 \\ 0 & 0 & 1-\hat{b}_4 & 1-2\hat{b}_4 \\ 0 & 0 & 0 & (1-\hat{b}_4)^2 \end{bmatrix} =
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{(1-\hat{b}_4)^4} \left[ \begin{array}{cccc} (1-\hat{b}_4)(\hat{\sigma}_{11})^2 & (1-\hat{b}_4)\hat{\sigma}_{12} & (1-\hat{b}_4)\hat{\sigma}_{13} & (1-\hat{b}_4)\hat{\sigma}_{14} \\ (1-\hat{b}_4)\hat{\sigma}_{21} & (1-\hat{b}_4)(\hat{\sigma}_{22})^2 & (1-\hat{b}_4)\hat{\sigma}_{23} & (1-\hat{b}_4)\hat{\sigma}_{24} \\ (1-\hat{b}_4)\hat{\sigma}_{31} & (1-\hat{b}_4)\hat{\sigma}_{32} & (1-\hat{b}_4)(\hat{\sigma}_{33})^2 & (1-\hat{b}_4)\hat{\sigma}_{34} \\ \hat{c}_{41} & \hat{c}_{42} & \hat{c}_{43} & \hat{c}_{44} \end{array} \right] \times \\
&\quad \text{with:} \\
&\quad \hat{c}_{41} = (1-2\hat{b}_4) \left[ (\hat{\sigma}_{11})^2 + \hat{\sigma}_{21} + \hat{\sigma}_{31} \right] + (1-\hat{b}_4)^2 \hat{\sigma}_{41} \\
&\quad \hat{c}_{42} = (1-2\hat{b}_4) \left[ \hat{\sigma}_{12} + (\hat{\sigma}_{22})^2 + \hat{\sigma}_{32} \right] + (1-\hat{b}_4)^2 \hat{\sigma}_{42} \\
&\quad \hat{c}_{43} = (1-2\hat{b}_4) \left[ \hat{\sigma}_{13} + \hat{\sigma}_{23} + (\hat{\sigma}_{33})^2 \right] + (1-\hat{b}_4)^2 \hat{\sigma}_{43} \\
&\quad \hat{c}_{44} = (1-2\hat{b}_4) \left[ \hat{\sigma}_{14} + \hat{\sigma}_{24} + \hat{\sigma}_{34} \right] + (1-\hat{b}_4)^2 (\hat{\sigma}_{44})^2 \\
&\quad \times \begin{bmatrix} 1-\hat{b}_4 & 0 & 0 & 1-2\hat{b}_4 \\ 0 & 1-\hat{b}_4 & 0 & 1-2\hat{b}_4 \\ 0 & 0 & 1-\hat{b}_4 & 1-2\hat{b}_4 \\ 0 & 0 & 0 & (1-\hat{b}_4)^2 \end{bmatrix} = \\
&= \frac{1}{(1-\hat{b}_4)^4} \left[ \begin{array}{cccc} (1-\hat{b}_4)^2 (\hat{\sigma}_{11})^2 & (1-\hat{b}_4)^2 \hat{\sigma}_{12} & (1-\hat{b}_4)^2 \hat{\sigma}_{13} & \hat{d}_{14} \\ (1-\hat{b}_4)^2 \hat{\sigma}_{21} & (1-\hat{b}_4)^2 (\hat{\sigma}_{22})^2 & (1-\hat{b}_4)^2 \hat{\sigma}_{23} & \hat{d}_{24} \\ (1-\hat{b}_4)^2 \hat{\sigma}_{31} & (1-\hat{b}_4)^2 \hat{\sigma}_{32} & (1-\hat{b}_4)^2 (\hat{\sigma}_{33})^2 & \hat{d}_{34} \\ \hat{d}_{41} & \hat{d}_{42} & \hat{d}_{43} & \hat{d}_{44} \end{array} \right], \text{ with} \\
&\quad \hat{d}_{14} = (1-\hat{b}_4) (1-2\hat{b}_4) \left[ (\hat{\sigma}_{11})^2 + \hat{\sigma}_{12} + \hat{\sigma}_{13} \right] + (1-\hat{b}_4)^3 \hat{\sigma}_{14} \\
&\quad \hat{d}_{24} = (1-\hat{b}_4) (1-2\hat{b}_4) \left[ \hat{\sigma}_{21} + (\hat{\sigma}_{22})^2 + \hat{\sigma}_{23} \right] + (1-\hat{b}_4)^3 \hat{\sigma}_{24} \\
&\quad \hat{d}_{34} = (1-\hat{b}_4) (1-2\hat{b}_4) \left[ \hat{\sigma}_{31} + \hat{\sigma}_{32} + (\hat{\sigma}_{33})^2 \right] + (1-\hat{b}_4)^3 \hat{\sigma}_{34} \\
&\quad \hat{d}_{41} = (1-\hat{b}_4) (1-2\hat{b}_4) \left[ (\hat{\sigma}_{11})^2 + \hat{\sigma}_{21} + \hat{\sigma}_{31} \right] + (1-\hat{b}_4)^3 \hat{\sigma}_{41} \\
&\quad \hat{d}_{42} = (1-\hat{b}_4) (1-2\hat{b}_4) \left[ \hat{\sigma}_{12} + (\hat{\sigma}_{22})^2 + \hat{\sigma}_{32} \right] + (1-\hat{b}_4)^3 \hat{\sigma}_{42} \\
&\quad \hat{d}_{43} = (1-\hat{b}_4) (1-2\hat{b}_4) \left[ \hat{\sigma}_{13} + \hat{\sigma}_{23} + (\hat{\sigma}_{33})^2 \right] + (1-\hat{b}_4)^3 \hat{\sigma}_{43} \\
&\quad \hat{d}_{44} = (1-\hat{b}_4)^2 (1-2\hat{b}_4) \left[ \hat{\sigma}_{41} + \hat{\sigma}_{42} + \hat{\sigma}_{43} + \hat{\sigma}_{14} + \hat{\sigma}_{24} + \hat{\sigma}_{34} \right] + \\
&\quad + (1-2\hat{b}_4)^2 \left[ (\hat{\sigma}_{11})^2 + \hat{\sigma}_{21} + \hat{\sigma}_{31} + \hat{\sigma}_{12} + (\hat{\sigma}_{22})^2 + \hat{\sigma}_{32} + \hat{\sigma}_{13} + \hat{\sigma}_{23} + (\hat{\sigma}_{33})^2 \right] + \\
&\quad + (1-\hat{b}_4)^4 (\hat{\sigma}_{44})^2 = \\
&\quad = (1-\hat{b}_4)^2 (1-2\hat{b}_4) 2 \left[ \hat{\sigma}_{14} + \hat{\sigma}_{24} + \hat{\sigma}_{34} \right] + \\
&\quad + (1-2\hat{b}_4)^2 \left[ (\hat{\sigma}_{11})^2 + (\hat{\sigma}_{22})^2 + (\hat{\sigma}_{33})^2 + 2(\hat{\sigma}_{12} + \hat{\sigma}_{13} + \hat{\sigma}_{23}) \right] + \\
&\quad + (1-\hat{b}_4)^4 (\hat{\sigma}_{44})^2.
\end{aligned}$$

The *square root* of the analytical expressions for the *diagonal* elements of the variance-covariance matrix derived above was then used in our programs<sup>25</sup> to compute the *standard errors* of the structural

<sup>25</sup>These EViews programs are available upon request.

parameters of interest,  $\beta_{0,+k} \equiv \hat{\beta}_1$ ,  $\beta_{\pi,+k} \equiv \hat{\beta}_2$ ,  $\beta_{x,+q} \equiv \hat{\beta}_3$  and  $\beta_{i,-1} \equiv \hat{\beta}_4$ , reported in our tables in Appendix A in the notation we established in the present paper.

**7.3. The Delta Method: Application to Backward-Looking Taylor Rules.** Recovering the *long-run* policy response parameters, which is often of interest – e.g., in Nelson (2000, 2001, 2003) – out of the *directly* estimated *short-run* parameters in *backward-looking* Taylor rules and computing delta method *standard errors* for the former can be explained by analogy to what we already did for our forward-looking rules.<sup>26</sup>

Now we estimated directly:

$$i_t = b_{0,-1} + b_{\pi,-1}\pi_{t-1} + b_{x,-1}x_{t-1} + b_{i,-1}i_{t-1} + e_t.$$

In a long-run equilibrium,

$$i_t = i_{t-1} = i,$$

so that

$$i = b_{0,-1} + b_{\pi,-1}\pi_{t-1} + b_{x,-1}x_{t-1} + b_{i,-1}i,$$

and solving for  $i$ , one obtains:

$$i = \underbrace{\frac{b_{0,-1}}{1 - b_{i,-1}}}_{\equiv \beta_{0,-1}} + \underbrace{\frac{b_{\pi,-1}}{1 - b_{i,-1}}}_{\equiv \beta_{\pi,-1}}\pi_{t-1} + \underbrace{\frac{b_{x,-1}}{1 - b_{i,-1}}}_{\equiv \beta_{x,-1}}x_{t-1},$$

with the respective *long-run* intercept  $\beta_{0,-1}$  and responses  $\beta_{\pi,-1}$  and  $\beta_{x,-1}$ , as defined above.

Re-defining  $\beta_{0,-1} \equiv \beta_1$ ,  $\beta_{\pi,-1} \equiv \beta_2$ ,  $\beta_{x,-1} \equiv \beta_3$ ,  $b_{0,-1} \equiv b_1$ ,  $b_{\pi,-1} \equiv b_2$ ,  $b_{x,-1} \equiv b_3$  and  $\beta_{i,-1} \equiv b_{i,-1} \equiv b_4 \equiv \beta_4$  like we did in the preceding subsection, for an easier matrix manipulation, results in the *same* derivation for the computation of standard errors in the case of *four* relevant coefficients, as appropriate here.<sup>27</sup> Accordingly, our long-run responses and their standard errors from *backward-looking* Taylor rules were computed using the *same* formulas but *different* numerical values (i.e., from *different* estimated equations) as those for the forward-looking rules, provided that the number of parameters of interest (hence, the rank of the matrices involved in the delta method) is 4.<sup>28</sup>

<sup>26</sup>We do not report, however, such long-run responses and their delta method standard errors for the regressions in Table 4 because of the uninterpretable results (due to a likely misspecification, as the coefficients to the lagged dependent variable indicate, especially for the period of operational independence).

<sup>27</sup>We have not derived and computed delta method standard errors for the version of our forward-looking equations with an explicit exchange rate term (in Table 11) and, hence, a rank of 5 for the matrices to be manipulated as shown above, again because of regression results which are difficult to interpret (due to a likely misspecification, as commented earlier).

<sup>28</sup>Our EViews programs are available upon request.

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